

Here $A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$

a) $(A+B)^T = \begin{bmatrix} 7 & -5 \\ -1 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & -1 \\ -5 & 7 \end{bmatrix}$.

$$A^T + B^T = \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ -5 & 7 \end{bmatrix}.$$

Thus $(A+B)^T = A^T + B^T$.

b) $(AB)^T = \left(\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 \\ 1 & 14 \end{bmatrix}$

$$B^T A^T = \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -7 & 14 \end{bmatrix}$$

Thus $(AB)^T = B^T A^T$.

(we can check $(AB)^T \neq A^T B^T$)

Take $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

(Note A, B are not zero matrices)

$$A \cdot B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \text{ a zero matrix.}$$

Thus, a product of two matrices can be zero with none of the matrices being zero matrix themselves.