The given set is a basis for a subspace W. Use the Gram-Schmidt process to produce an orthogonal basis for W.

$$\begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix}$$

Solution: Let's denote the given vectors as

$$\vec{x_1} = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{x_2} = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix}, \quad \vec{x_3} = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix}$$

Then an orthogonal basis is $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ where we find them as follows:

$$\vec{v}_1 = \vec{x}_1 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix},$$

$$\vec{v}_2 = \vec{x}_2 - \left(\frac{\vec{x}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1}\right)\vec{v}_1 = \vec{x}_2 - (-3)\vec{v}_1 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix} + 3\begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix},$$

$$\vec{v}_3 = \vec{x}_3 - \left(\frac{\vec{x}_3 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1}\right)\vec{v}_1 - \left(\frac{\vec{x}_3 \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2}\right)\vec{v}_2$$

$$= \vec{x}_3 - \frac{1}{2}\vec{v}_1 - \frac{5}{2}\vec{v}_2 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} - \frac{5}{2}\begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}.$$