Nul A, Col A, Row A, Rank

1. If a 6×3 matrix A has rank 3, find $\dim \mathrm{Nul} A, \dim \mathrm{Row}\, A$ and rank A^T.

2. Suppose a 5×6 matrix A has four pivot columns. What is $\dim \mathrm{Nul} A$? Is $\mathrm{Col} A = \mathbb{R}^4$? Why or why not?

3. If the null space of a 7×6 matrix A is 5 -dimensional, what is the dimension of the column space of A?

4. If the null space of a 5×6 matrix A is 4 -dimensional, what is the dimension of the row space of A?

5. If A is a 4×3 matrix, what is the largest possible dimension of the row space of A ? If A is a 3×4 matrix, what is the largest possible dimension of the row space of A? Explain.

6. If A is a 6×4 matrix, what is the smallest possible dimension of Nul A?

7. Could a 6×9 matrix have a two-dimensional null space?