Note that $\dim \mathbb{P}_3 = 4$.

We have the set, $H = \{1, 1-t, 2-4t+t^2, 6-18t+9t^2-t^3\}$.

Since there are 4 vectors, all we need to show is H is a L.I. set. (Then H is a basis for $\mathbb{P}_3$).

Now let

$$c_1 \cdot 1 + c_2(1-t) + c_3(2-4t+t^2) + c_4(6-18t+9t^2-t^3) = 0$$

$$\Rightarrow \quad c_1 + c_2 - c_2 t + 2c_3 - 4c_3 t + c_3 t^2 + 6c_4 - 18c_4 t + 9c_4 t^2 - c_4 t^3 = 0$$

or, $$(c_1 + c_2 + 2c_3 + 6c_4) + (-c_2 - 4c_3 - 18c_4)t + (c_3 + 9c_4)t^2 + (-c_4)t^3 = 0 + 0 \cdot t + 0 \cdot t^2 + 0 \cdot t^3$$

$$\Rightarrow \text{solve} \begin{cases} c_1 + c_2 + 2c_3 + 6c_4 = 0 & c_1 = 0 \\ -c_2 - 4c_3 - 18c_4 = 0 & -c_2 = 0 \text{ or } c_2 = 0 \\ c_3 + 9c_4 = 0 & c_3 + 0 = 0 \text{ or } c_3 = 0 \\ -c_4 = 0 & \text{or, } c_4 = 0 \end{cases}$$

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Thus $c_1 = c_2 = c_3 = c_4 = 0$, the trivial solution.

$\Rightarrow H$ is a L.I. set.

So, the polynomials form a basis of $\mathbb{P}_3$.