Diagonalization: short-answer questions

1. Without calculation check if each of the following matrices is diagonalizable.

$$\begin{bmatrix} 2 & 0 & 0 \\ -3 & 4 & 0 \\ 5 & 4 & -1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$$

2. A is a 5×5 matrix with two eigenvalues. One eigenspace is three-dimensional, and the other eigenspace is two dimensional. Is A diagonalizable? Why?

3. A is a 3×3 matrix with two eigenvalues. Each eigenspace is one-dimensional. Is A diagonalizable? Why?

4. A is a 4×4 matrix with three eigenvalues. One eigenspace is one-dimensional, and one of the other eigenspaces is two-dimensional. Is it possible that A is not diagonalizable? Justify your answer.

5. Construct a nonzero 2×2 matrix that is invertible but not diagonalizable.

6. Construct a nondiagonal 2×2 matrix that is diagonalizable but not invertible.