Mechanical and Electrical Vibrations

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This illustrates a fundamental relationship between mathematics and physics: **many physical problems may have the same mathematical model.**

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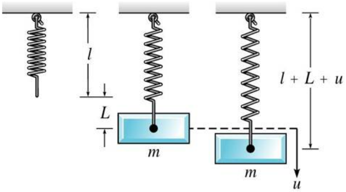

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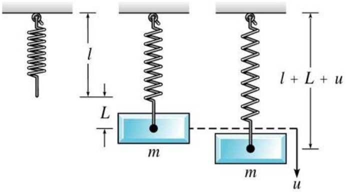

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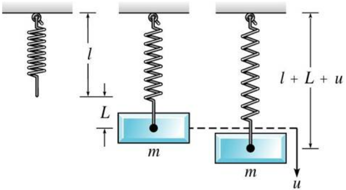

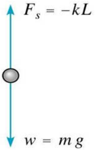

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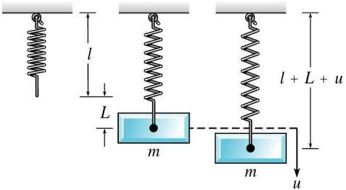

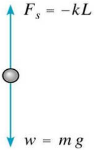

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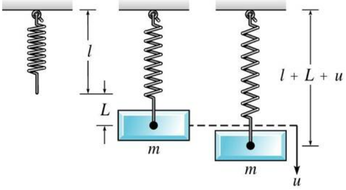

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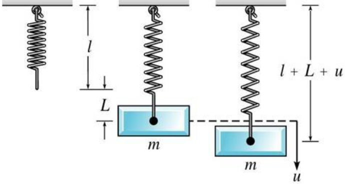

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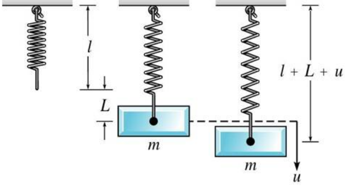

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In this dynamic problem there are now four separate forces that must be considered as given in the next page.

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4. External forces: An applied external force $F(t)$. This could be a force due to the motion of the mount to which the spring is attached, or it could be a force applied directly to the mass. Often the external force. is periodic.

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The complete formulation of the vibration problem requires that we specify two initial conditions,namely, the initial position u_0 and the initial velocity v_0 of the mass:

$$u(0) = u_0, \qquad u'(0) = v_0.$$

We have the formulas:

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spring constant: $k = \frac{mg}{L}$.

Example 1

A mass weighing 4 lb stretches a spring 2 in. Suppose that the mass is given an additional 6 in displacement in the positive direction and then released. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has a velocity of 3 ft/s. Under the assumptions discussed in this section, formulate the initial value problem that governs the motion of the mass.

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