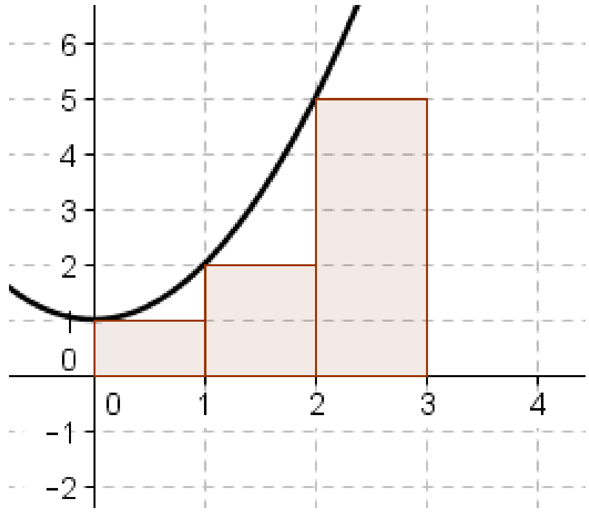
1. Consider the definite integral $\int_0^3 x^2 + 1\, dx$. Approximate the integral using a Riemann sum with $n = 3$ using **left** endpoints.

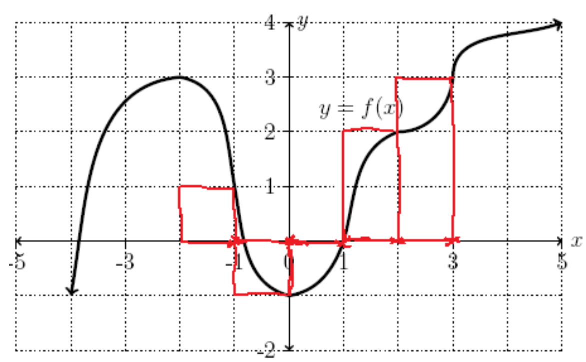
   We graph the function $f(x) = x^2 + 1$, then divide the interval $[0, 3]$ into three subintervals, and draw rectangles using end points: 0, 1, 2 as shown.



   Width of each rectangle, $\Delta x = 1$.

   So $R_{L_3} \approx \Delta x\,[f(0) + f(1) + f(2)] = 1(1 + 2 + 5) = 8$.

2. Consider the graph of $y = f(x)$ given below. (a) Divide the closed interval $[-2, 3]$ into 5 equal subintervals and draw the corresponding rectangles using the right endpoints of each subinterval. (b) Find the Riemann sum for $y = f(x)$ on the interval $[-2, 3]$ for $n = 5$, taking the sample points to be right endpoints.



   Width of each rectangle, $\Delta x = 1$.

   So $R_{R_5} \approx \Delta x\,[f(-1) + f(0) + f(1) + f(2) + f(3)] = 1(1 + (-1) + 0 + 2 + 3) = 5$.

   (c) $\int_{-2}^{3} f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$, where $\Delta x = \frac{3-(-2)}{n} = \frac{5}{n}$, and x_i is a sample point.