(6.1) Area between Two Curves

Full Name: ______________________________

1. Sketch the region enclosed by the graphs of the given equations. Then, use a definite integral to find the exact value of the area of the region.

   (a) Note that $y = 0$ define the x-axis. Hence, we are simply looking for the area between $y = \dfrac{1}{x}$ and the x-axis on the interval $[1, e]$.

   (b) Similar to (a) but with $y = \sin x$ and the interval $\left[\dfrac{\pi}{6}, \dfrac{\pi}{2}\right]$.

   (c) Similar to (a) but with $y = \dfrac{1}{1+x^2}$ and the interval $\left[\dfrac{\sqrt{3}}{3}, 1\right]$.

   (d) Note that we are dealing with a function of y in this case. Fortunately, one of our curves if $x = 0$, namely the y-axis. So, this problem is analogous to (a), but with the function $x = e^y$ on the interval of the y-axis $[1 , \ln 2]$.

   (e) To find the bounds on our region, we need to set the two functions equal to one another. Upon finding the appropriate x-values to define our interval of interest, we determine which function is on top and compute $\displaystyle\int_a^b (\text{top function} - \text{bottom function})\, dx$.

   (f) Similar to (e) .

   (g) Similar to (e) and (f) , but with two distinct area regions to consider as a result of there being three intersection points between these two curves. Within each area region, we will need to establish which function is top and which is bottom (it could switch from region to region).

   (h) This is analogous to (e) and (f) , albeit with functions of y instead of functions of x . As such, finding intersection points will involve solving for y-values, and our integral will be in the form $\displaystyle\int_a^b (\text{rightmost function of } y \ - \text{ leftmost function of } y)\, dy$.