(5.5) The Substitution Rule

Full Name: ______________________________

1. Compute each of the following integrals.

   (a) Are $\sec^2 t$ and $\tan t$ related by way of derivative? If so, which needs to be our u in order that the other be within the resulting expression for du?

   (b) We don't have a $\sin(\cdots)$ to pair with our $\cos(\cdots)$ through a relationship by way of derivative, but can we pair $e^x + 2$ and e^x through such a relationship?

   (c) The polynomials $1 + x^4$ and x^3 differ by one degree. Perhaps there is a derivative relationship here? Which one needs to be our u in order that the other be within the resulting expression for du?

   (d) The polynomials $2x^3$ and $24x^2$ differ by one degree. Perhaps we could use derivative to relate them?!

   (e) The polynomials $12x - 10$ and $3x^2 - 5x$ differ by one degree. Perhaps there is a derivative relationship here?

   (f) Note the polynomials $2x$ and $1 + x^2$.

   (g) We have a $\sin(\cdots)$ to pair with our $\cos(\cdots)$ through a relationship by way of derivative. One needs to play the role of u and the other will result in the expression of du. Test to see which candidate for u will make for a cleaner substitution.

   (h) What if we rewrote the expression of the function inside our integral as $(\ln x)^3 \, \frac{1}{x} \, dx$. Perhaps this makes it more apparent what we need u to be?!

   (i) This (and the next two examples) are a subtle application of u-substitution. Do recall that any polynomial of the form $mx + b$ has a derivative of m. This can be useful, since letting $u = mx + b$, it follows that $du = m\,dx$.

   (j) See hint for (i) above.

   (k) See hint for (i) above. Add the additional observation that we need to compute $F(b) - F(a)$. In short, we can forget the bounds until we have our $F(x)$ function (as we did in every problem previous from this sheet). Once we have our $F(x)$, we simply plug in 5, plug in 0, and subtract the results.