(5.4) Indefinite Integrals & Net Change Theorem

Name: ______________________________

1. Compute each of the following indefinite integrals
   (a) Recall that $\sqrt{x}$ can be rewritten as $\sqrt[2]{x}$. Also recall that every radical expression of the form $\sqrt[m]{x}$ can be rewritten as $x^{1/m}$. From there, apply the power rule: $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$.

   (b) Recall that $\frac{1}{x^m}$ can be rewritten as x^{-m}. In general, moving any exponential factor from one side of the fraction bar to the other changes the sign on the exponent.

   (c) Note the hint from (a), and also note that $\frac{A+B+C}{D}$ is equal to $\frac{A}{D} + \frac{B}{D} + \frac{C}{D}$. Given that we have no quotient rule for integrals, this fact can be applied to our advantage here, as the polynomial in the denominator with only one term simplifies nicely when divided from each of the terms in the numerator.

   (d) We have no product rule for antiderivatives. Why not multiply the polynomials in parentheses first and then try to apply what you know of calculus to compute the integral.

   (e) Regarding the denominator, see the hint from (c). Regarding the numerator, recall that $(2x+3)^2$ is equal to $(2x+3)(2x+3)$ and see the hint from (d).

   (f) It looks like some of the factors in the numerator and denominator can cancel. What happens after you simplify this function algebraically? Will calculus now nicely apply to compute the integral?

   (g) Recall the fact that $\cos^2\theta + \sin^2\theta$ is equal to 1. Moving the $\sin^2\theta$ term to the other side of the equation, we would have that $\cos^2\theta = 1 - sin^2\theta$. Note the original problem and observe how this fact will be very useful here.

2. Compute each of the following definite integrals
   (a) Observe that the one factor of $1 + y^2$ in the numerator will cancel with one of the factors of $1 + y^2$ from the denominator. At this point, you will need an integration fact to find $F(x)$.

   (b) Observe that the function within this integral can be rewritten as $\frac{(1-u^2)^{1/2}}{1-u^2}$.

   (c) Note that $(e^x)^3$ can be expressed as e^{3x}. As such, the exponential factors in the numerator and denominator can subtract nicely. At that point, finding $F(x)$ follows directly from a known integration fact.

   (d) Note the polynomial in the denominator has two terms (unlike the one term in (1c) and (1e) above). As such, we will have to be a bit more careful here in order to divide away the numerator accurately. Note that the denominator can be written as $2(2x-1)$. Should the numerator polynomial also have a factor of $2x-1$, we would be in business (and could cancel straight away). Does the numerator polynomial have a factor of $2x-1$?