Worksheet 7.8

Full Name: ________________ Score: ________

1. Determine whether each of the following improper integrals converges or diverges. Find the exact values of those that converge.

(a) $\int_2^\infty \frac{8}{x^5}\,dx$ $= \lim_{t\to\infty} \int_2^t \frac{8}{x^5}dx = \lim_{t\to\infty}\left(\frac{-2}{x^4} \Big|_2^t \right)$

$= \lim_{t\to\infty}\left(\frac{-2}{t^4} + \frac{2}{16}\right)$

$= \frac{2}{16} - 0 = \frac{2}{16} = 1/8$

the improper integral converges to 1/8.

(b) $\int_1^\infty \frac{2}{3x+1}\,dx$ $= \lim_{t\to\infty} \int_1^t \frac{2}{3x+1}dx = \lim_{t\to\infty}\left(\frac{2}{3}\ln|3x+1| \Big|_1^t \right)$

$= \lim_{t\to\infty}\left(\frac{2}{3}\ln|3t+1| - \frac{2}{3}\ln|4|\right)$

$= \infty$

The integral diverges.

(c) $\int_3^\infty \frac{1}{(4x-3)^{\frac{3}{2}}}\,dx$ $= \lim_{t\to\infty} \int_3^t \frac{1}{(4x-3)^{3/2}}dx$

$= \lim_{t\to\infty}\left(\frac{1}{4}(-2)\frac{1}{\sqrt{4x-3}} \Big|_3^t \right)$

$= \lim_{t\to\infty}\left(\frac{1}{2\sqrt{9}} - \frac{1}{2\sqrt{4t-3}}\right) = \frac{1}{6}$.

The integral converges to 1/6.

(d) $\int_0^4 \frac{2}{\sqrt{x}}\,dx = \lim_{t\to 0^+} \int_t^4 \frac{2}{\sqrt{x}}\,dx$

$$= \lim_{t\to 0^+} \left(4\sqrt{x}\Big|_t^4 \right)$$

$$= \lim_{t\to 0^+} \left(8 - 4\sqrt{t} \right) = \underline{\underline{8}}$$

(e) $\int_0^2 \frac{2}{(x-2)^2}\,dx = \lim_{t\to 2^-} \int_0^t \frac{2}{(x-2)^2}\,dx$

$$= \lim_{t\to 2^-} \left(\frac{-2}{x-2}\Big|_0^t \right)$$

$$= \lim_{t\to 2^-} \left(-\frac{2}{t-2} + \frac{2}{0-2} \right) = +\infty$$

$\int_0^2 \frac{2}{(x-2)^2}\,dx$ diverges.

(f) $\int_1^3 \frac{1}{x-2}\,dx = \int_1^2 \frac{1}{x-2}\,dx + \int_2^3 \frac{1}{x-2}\,dx$ (two improper integrals)

$$\int_2^3 \frac{1}{x-2}\,dx = \lim_{t\to 2^+} \int_t^3 \frac{1}{x-2}\,dx$$

$$= \lim_{t\to 2^+} \left(\ln|x-2| \Big|_t^3 \right)$$

$$= \lim_{t\to 2^+} \left(\ln|1| - \ln|t-2| \right)$$

$$= 0 - (-\infty) = \infty.$$

$\int_2^3 \frac{1}{x-2}\,dx$ is divergent hence $\int_1^3 \frac{1}{x-2}\,dx$ is divergent

Note: If you compute $\int_1^2 \frac{1}{x-2}\,dx$ you would get $-\infty$ which is also divergent.