Worksheet 7.1 & 7.2

Full Name: ______________________________ Score: ________

1. Use integration by parts to evaluate each of the following indefinite integrals.

(a) $\int x \cos x \, dx$

Let $u = x$, $dv = \cos x \, dx$

$du = dx$, $v = \sin x$

$\int x \cos x \, dx = \int u \, dv = uv - \int v \, du$

$= x \sin x - \int \sin x \, dx$

$= x \sin x + \cos x + C$

(b) $\int y e^y \, dy$

Let $u = y$, $dv = e^y dy$

$du = dy$, $v = e^y$

$\int y e^y dy = \int u \, dv = uv - \int v \, du$

$= y e^y - \int e^y dy$

$= y e^y - e^y + C$ //

(c) $\int t^{15} \ln t \, dt$

Let $u = \ln t$, $dv = t^{15}$

$du = \frac{1}{t} dt$, $v = \frac{t^{16}}{16}$

$\int t^{15} \ln t \, dt = \frac{t^{16}}{16} \ln t - \int \frac{t^{\cancel{16}\,15}}{16} \frac{1}{\cancel{t}} dt$

$= \frac{t^{16}}{16} \ln t - \frac{1}{16} \int t^{15} dt = \frac{t^{16}}{16} \ln t - \frac{1}{(16)^2} t^{16} + C$

(d) $\int x^2 \sin 2x \, dx$

Let $u = x^2$, $dv = \sin 2x \, dx$

$du = 2x\,dx$, $v = \frac{-\cos(2x)}{2}$

$\int x^2 \sin 2x \, dx = -\frac{x^2}{2}\cos(2x) - \int -\frac{\cos(2x)}{\cancel{2}} \cdot \cancel{2}x\,dx$

$= -\frac{x^2}{2}\cos 2x + \int x\cos(2x)\,dx = \boxed{-\frac{x^2}{2}\cos(2x) + \frac{x}{2}\sin 2x + \frac{\cos(2x)}{4} + C}$

Let $u = x$, $dv = \cos 2x \, dx$

$du = dx$, $v = \frac{\sin(2x)}{2}$

$\int x\cos 2x\,dx = \frac{x}{2}\sin 2x - \int \frac{\sin 2x}{2}\,dx$

$= \frac{x}{2}\sin 2x + \frac{\cos(2x)}{4} + c$

(e) $\int x^3 e^x \, dx$

Let $u = x^3$, $dv = e^x dx$

$du = 3x^2 dx$, $v = e^x$

$\int x^3 e^x dx = x^3 e^x - 3\int x^2 e^x dx$

$\int x^2 e^x dx$ Let $u = x^2$; $dv = e^x dx$

$du = 2x\,dx$, $v = e^x$

$= x^2 e^x - 2\int x e^x dx$

$= x^2 e^x - 2(xe^x - e^x) + c_2$

$\int x e^x dx$, Let $u = x$, $dv = e^x dx$

$\Rightarrow du = dx$, $v = e^x$

$= xe^x - \int e^x dx = xe^x - e^x + c_1$

$\int x^3 e^x dx = x^3 e^x - 3\left(x^2 e^x - 2(xe^x - e^x)\right) + C$

$= x^3 e^x - 3(x^2 e^x - 2xe^x + 2e^x) + C$

$= x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + C$

(f) $\int \ln 12x \, dx$

Let $u = \ln(12x)$ $\quad dv = dx$

$\ln(12x) = \ln 12 + \ln x$

$du = \frac{1}{x}$, $\quad v = x$

$$\int \ln(12x)\,dx = x\ln(12x) - \int \cancel{x}\cdot\frac{1}{\cancel{x}}\,dx$$

$$= x\ln(12x) - x + C$$

(g) $\int \tan^{-1}(y)\,dy$

Let $u = \tan^{-1} y$, $\quad dv = dy$

$du = \frac{1}{1+y^2}\,dy$, $\quad v = y$

$$\int \tan^{-1} y\,dy = y\tan^{-1} y - \int \frac{y}{1+y^2}\,dy$$

Let $u = 1+y^2 \Rightarrow du = 2y\,dy$

$\Rightarrow dy = \frac{du}{2y}$

$$\int \frac{y}{1+y^2}\,dy = \int \frac{\cancel{y}}{u}\,\frac{du}{2\cancel{y}}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(1+y^2) + C$$

$$\int \tan^{-1} y\,dy = y\tan^{-1} y - \frac{1}{2}\ln(1+y^2) + C$$

(h) $\int \arcsin(x)\,dx$

$u = \arcsin x$ $\quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$, $\quad v = x$

$$\int \arcsin x\,dx = x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

Let $u = 1-x^2$

$du = -2x\,dx \Rightarrow dx = \frac{-du}{2x}$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = \int \frac{\cancel{x}}{\sqrt{u}}\,\frac{-du}{2\cancel{x}}$$

$$= \int \frac{-1}{2\sqrt{u}}\,du = \sqrt{u} + C$$

$$= \sqrt{1-x^2} + C$$

$$\int \arcsin x\,dx = x\arcsin x - \sqrt{1-x^2} + C$$

(i) $\int e^{5x} \sin 2x \, dx$

$u = e^{5x}$, $dv = \sin 2x \, dx$

$du = 5e^{5x} dx$, $v = \frac{-\cos(2x)}{2}$

$\int = -\frac{1}{2} e^{5x} \cos(2x) + \int \frac{5}{2} e^{5x} \cos(2x) dx$

$= -\frac{1}{2} e^{5x} \cos(2x) + \frac{5}{2} \int e^{5x} \cos 2x \, dx$

Let $u = e^{5x}$, $dv = \cos 2x \, dx$

$du = 5e^{5x} dx$, $v = \frac{\sin 2x}{2}$

$\int e^{5x} \cos(2x) dx = \frac{1}{2} e^{5x} \sin 2x - \frac{5}{2} \int e^{5x} \sin 2x \, dx$

<u>So:</u>

$\int e^{5x} \sin 2x \, dx = -\frac{1}{2} e^{5x} \cos(2x) + \frac{5}{2} \left(\frac{1}{2} e^{5x} \sin 2x - \frac{5}{2} \int e^{5x} \sin 2x \, dx \right)$

$= -\frac{1}{2} e^{5x} \cos 2x + \frac{5}{4} e^{5x} \sin 2x - \frac{25}{4} \int e^{5x} \sin 2x \, dx$

$+ \frac{25}{4} \int e^{5x} \sin 2x \, dx$ $\qquad + \frac{25}{4} \int e^{5x} \sin 2x \, dx$

$\frac{29}{4} \int e^{5x} \sin 2x \, dx = -\frac{1}{2} e^{5x} \cos 2x + \frac{5}{4} e^{5x} \sin 2x + c,$

$\int e^{5x} \sin 2x \, dx = \frac{-2}{29} e^{5x} \cos 2x + \frac{5}{29} e^{5x} \sin 2x + C$

(j) $\int \cos x \sin^{12} x \, dx$

$$\int \sin^{12} x \cdot \cos x \, dx$$

$u = \sin x \Rightarrow du = \cos x \, dx$

$\Rightarrow dx = \frac{du}{\cos x}$

$$= \int u^{12} \cancel{\cos x} \frac{du}{\cancel{\cos x}}$$

$$= \int u^{12} du = \frac{u^{13}}{13} + c = \frac{\sin^{13} x}{13} + C.$$

(k) $\int \cos^3 x \sin^{10} x \, dx$ $= \int \cos^2 x \sin^{10} x \cos x \, dx$

$$= \int (1 - \sin^2 x) \sin^{10} x \cos x \, dx$$

Let $u = \sin x \Rightarrow du = \cos x \, dx$

$$\int \cos^3 x \sin^{10} x \, dx = \int (1 - u^2) u^{10} du$$

$$= \int u^{10} - u^{12} du = \frac{u^{11}}{11} - \frac{u^{13}}{13} + c$$

$$= \frac{\sin^{11} x}{11} - \frac{\sin^{13} x}{13} + C$$

Let $y = 3\theta \Rightarrow dy = 3\,d\theta$
$d\theta = dy/3$

$= \int \cos^{14} y \sin^5 y \frac{dy}{3} = \frac{1}{3}\int \cos^{14} y \sin^5 y\,dy$

(l) $\int \cos^{14}(3\theta)\sin^5(3\theta)\,d\theta$

$= \frac{1}{3}\int \cos^{14} y \sin^4 y \sin y\,dy$

$= \frac{1}{3}\int \cos^{14} y (\sin^2 y)^2 \sin y\,dy = \frac{1}{3}\int \cos^{14} y (1-\cos^2 y)^2 \sin y\,dy$

Let $u = \cos y \Rightarrow du = -\sin y\,dy$

$\frac{1}{3}\int \cos^{14} y (1-\cos^2 y)^2 \sin y\,dy = -\frac{1}{3}\int u^{14}(1-u^2)^2\,du$

$= \frac{-1}{3}\int u^{14}(u^4-2u^2+1)\,du = \frac{-1}{3}\int u^{18}-2u^{16}+u^{14}\,du = \frac{-1}{3}\left(\frac{u^{19}}{19} - 2\frac{u^{17}}{17} + \frac{u^{15}}{15} + C\right)$

$= \frac{-1}{3}\left(\frac{\cos^{19}(3\theta)}{19} - \frac{2\cos^{17}(3\theta)}{17} + \frac{\cos^{15}(3\theta)}{15}\right) + C$

(m) $\int \sec^2 x \tan^6 x\,dx$

$= \int \tan^6 x \sec^2 x\,dx$

Let $u = \tan x \Rightarrow du = \sec^2 x\,dx$

$\int \tan^6 x \sec^2 x\,dx = \int u^6\,du = \frac{u^7}{7} + C$

$= \frac{\tan^7 x}{7} + C$ //

(n) $\int \sec^5 x \tan^3 x\,dx = \int \sec^4 x \underset{\sec^2 x - 1}{\tan^2 x} \sec x \tan x\,dx$

$u = \sec x$
$du = \sec x \tan x\,dx$

$= \int \sec^4 x (\sec^2 x - 1) \sec x \tan x\,dx$

$= \int u^4(u^2-1)\,du = \int u^6 - u^4\,du = \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$ //