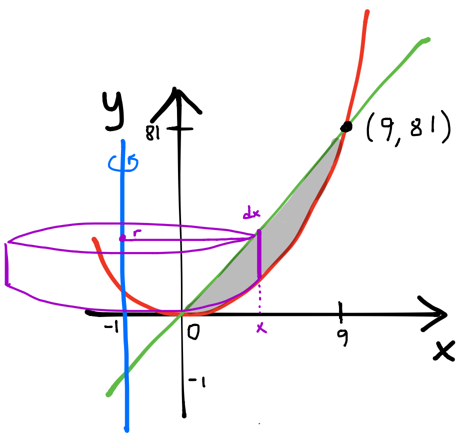
Worksheet 6.3

Full Name: ______________________________ Score: ________

1. Sketch the region enclosed by the graphs of the given equations. Then, use a definite integral to find the exact value of the volume of revolution obtained by rotating the region about the given axis of revolution.

   (a) $y = x^2$, $y = 9x$, about $x = -1$



SHELL METHOD

$r = x - (-1)$
$= x + 1$

$h = 9x - x^2$

$\text{Volume} = 2\pi \int_0^9 (x+1)(9x - x^2)\,dx$

WASHER METHOD

$y = x^2 \rightarrow x = \sqrt{y}$

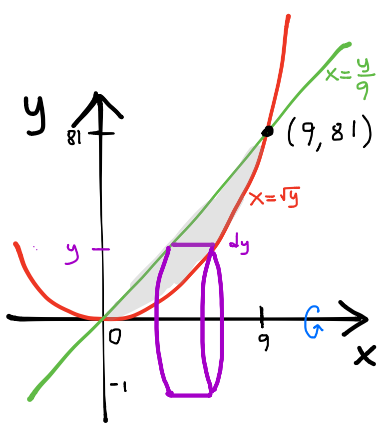
$y = 9x \rightarrow x = y/9$

$r_{outer} = \sqrt{y} - (-1)$
$= \sqrt{y} + 1$

$r_{inner} = \frac{y}{9} - (-1)$
$= \frac{y}{9} + 1$

$\text{Volume} = \pi \int_0^{81} (\sqrt{y}+1)^2 - (\frac{y}{9}+1)^2\,dy$

   (b) $y = x^2$, $y = 9x$, about the x-axis



SHELL METHOD:

$r = y - 0 = y$

$h = \sqrt{y} - \frac{y}{9}$

$\text{Volume} = 2\pi \int_a^b r\,h\,dy$

$\text{Volume} = 2\pi \int_0^{81} y\,(\sqrt{y} - \frac{y}{9})\,dy$

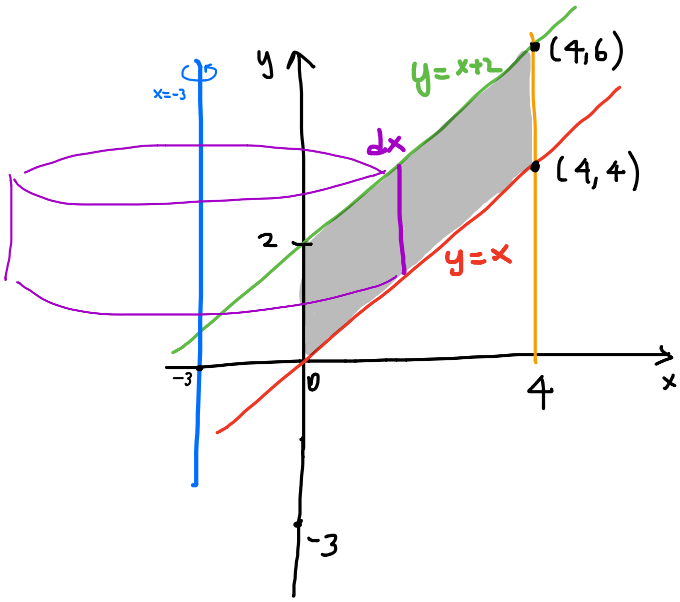
WASHER METHOD

$r_{outer} = 9x - 0$

$r_{inner} = x^2 - 0$

$\text{Volume} = \pi \int_0^9 (9x)^2 - (x^2)^2\,dx$

(c) $y = x, \quad y = x + 2, \quad x = 0, \quad x = 4$ about the $x = -3$



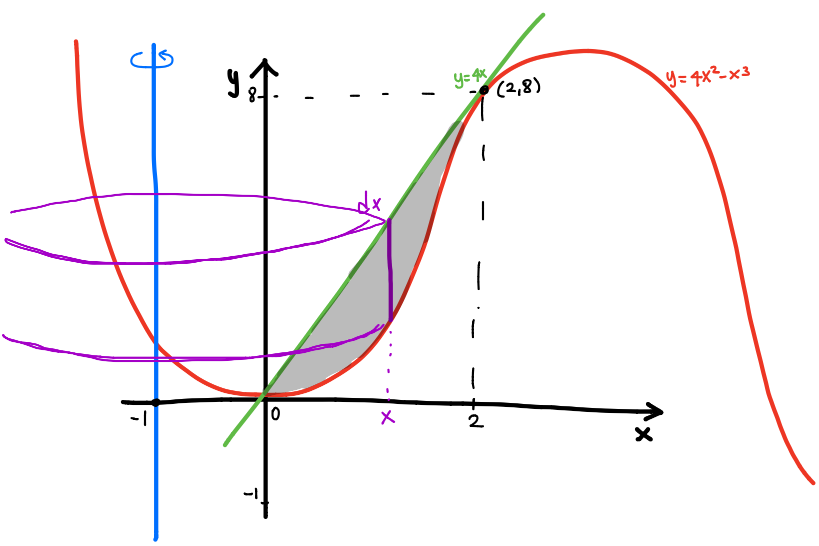
SHELL METHOD

$r = x - (-3) = x + 3$

$h = (x+2) - x = 2$

$\text{Volume} = 2\pi \int_0^4 (x+3) \cdot 2 \, dx$

(d) $y = 4x, \quad y = 4x^2 - x^3$ about the $x = -1$



SHELL METHOD:

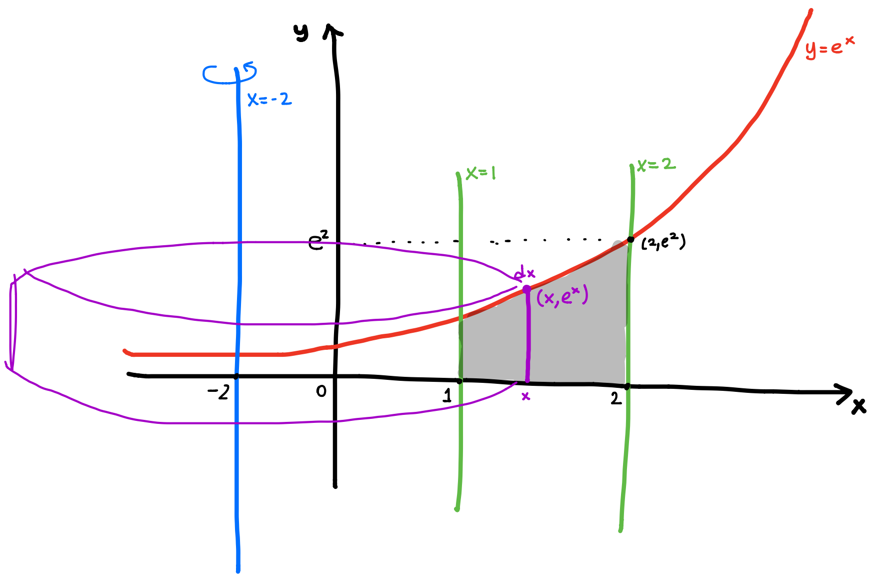
$r = x - (-1) = x + 1$

$h = 4x - (4x^2 - x^3)$

$= 4x - 4x^2 + x^3$

$\text{Volume} = 2\pi \int_0^2 (x+1)(4x - x^2 + x^3) \, dx$

(e) $y = e^x, \quad y = 0, \quad x = 1, \quad x = 2$ about $x = -2$



SHELL METHOD

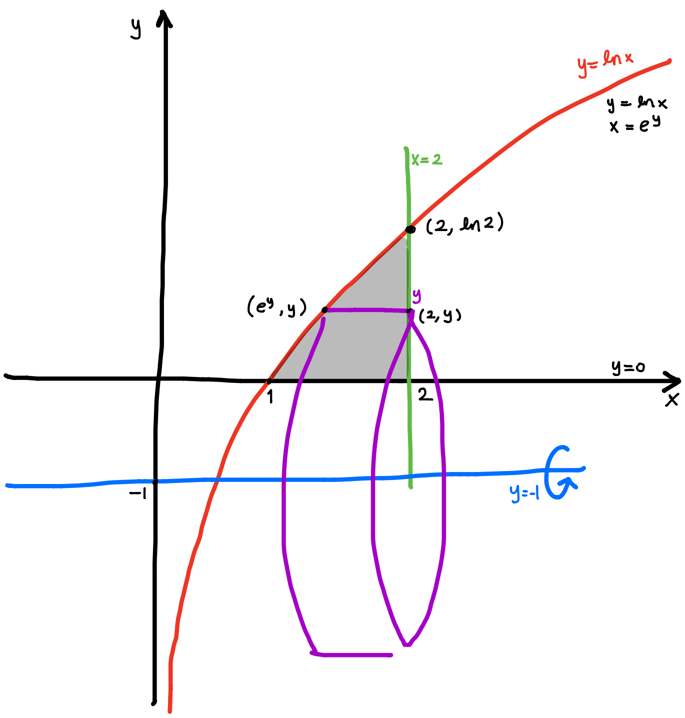
$r = x - (-2) = x+2$

$h = e^x - 0 = e^x$

$$\text{Volume} = 2\pi \int_1^2 (x+2)e^x \, dx$$

NOTE: This integral requires the method of integration by parts.

(f) $y = \ln x, \quad y = 0, \quad x = 2,$ about $y = -1$



SHELL METHOD

$r = y - (-1) = y+1$

$h = 2 - e^y$

$$V = 2\pi \int_0^{\ln 2} (y+1)(2-e^y)\,dy$$

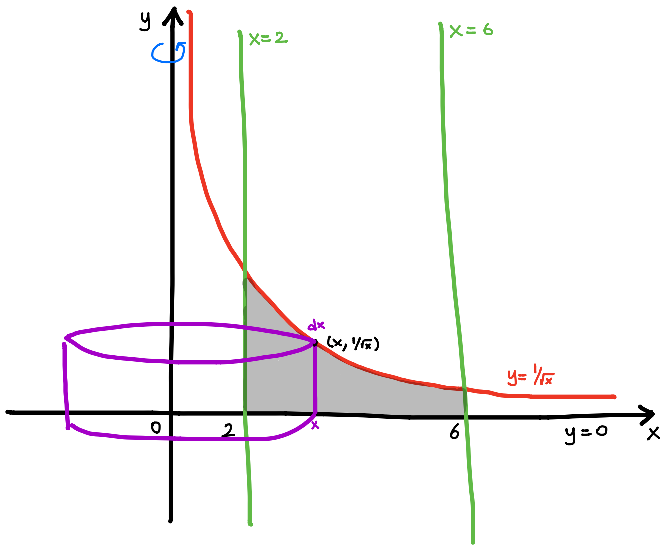
WASHER METHOD

$r_{outer} = \ln x - (-1) = 1 + \ln x$

$r_{inner} = 1$

$$V = \pi \int_1^2 (1+\ln x)^2 - 1^2 \; dx$$

(g) $y = \dfrac{1}{\sqrt{x}}, \quad y = 0, \quad x = 2, \quad x = 6$ about the y-axis



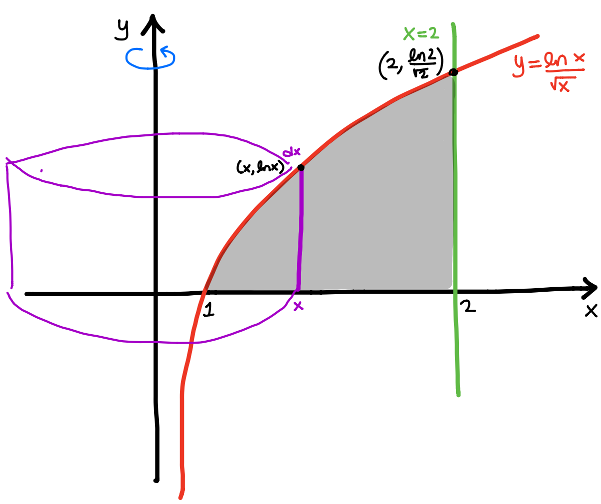
SHELL METHOD

$r = x - 0 = x$

$h = \frac{1}{\sqrt{x}} - 0 = \frac{1}{\sqrt{x}}$

$\text{Volume} = 2\pi \int_2^6 x \cdot \frac{1}{\sqrt{x}}\, dx$

(h) $y = \dfrac{\ln x}{\sqrt{x}}, \quad y = 0, \quad x = 2$, about the y-axis



SHELL METHOD

$r = x - 0 = x$

$h = \frac{\ln x}{\sqrt{x}} - 0$

$V = 2\pi \int_1^2 x \cdot \frac{\ln x}{\sqrt{x}}\, dx$