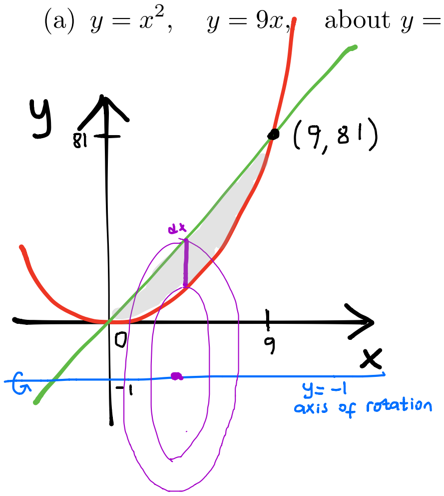
Worksheet 6.2

Full Name: ____ Score: ________

1. Sketch the region enclosed by the graphs of the given equations. Then, use a definite integral to find the exact value of the volume of revolution obtained by rotating the region about the given axis of revolution.

   (a) $y = x^2$, $y = 9x$, about $y = -1$



$r_{outer} = 9x - (-1) = 9x+1$

$r_{inner} = x^2 - (-1) = x^2+1$

WASHER METHOD:

$$\text{Volume} = \pi \int_0^9 (r_{outer}^2 - r_{inner}^2)\, dx$$

$$= \pi \int_0^9 (9x+1)^2 - (x^2+1)^2\, dx$$

SHELL METHOD:

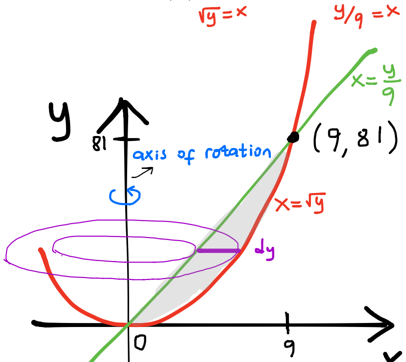
$$V = 2\pi \int_0^{81} (y+1)(\sqrt{y} - \tfrac{y}{9})\, dy$$

$r = y - (-1) = y+1$

$h = \sqrt{y} - \frac{y}{9}$

   (b) $y = x^2$, $y = 9x$, about the y-axis

$\sqrt{y} = x$ $\quad y/9 = x$



$r_{outer} = \sqrt{y} - 0 = \sqrt{y}$

$r_{inner} = \frac{y}{9} - 0 = \frac{y}{9}$

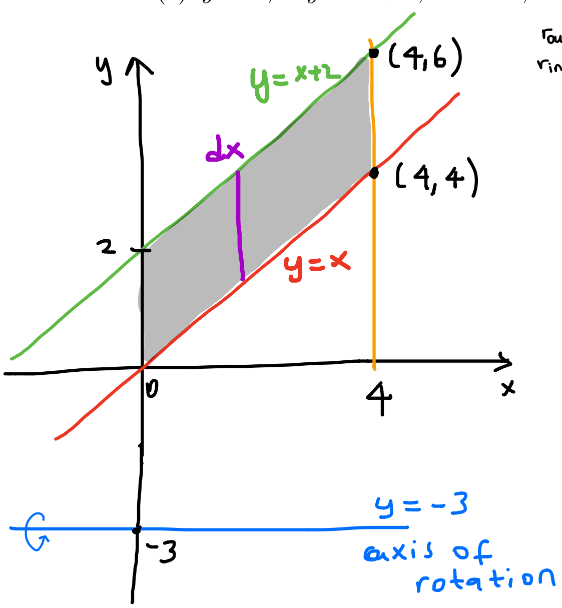
WASHER METHOD:

$$V = \pi \int_0^{81} \sqrt{y}^2 - \left(\frac{y}{9}\right)^2 dy$$

SHELL METHOD:

$$V = 2\pi \int_0^9 x \cdot (9x - x^2)\,dx$$

(c) $y = x, \quad y = x + 2, \quad x = 0, \quad x = 4$ about the $y = -3$



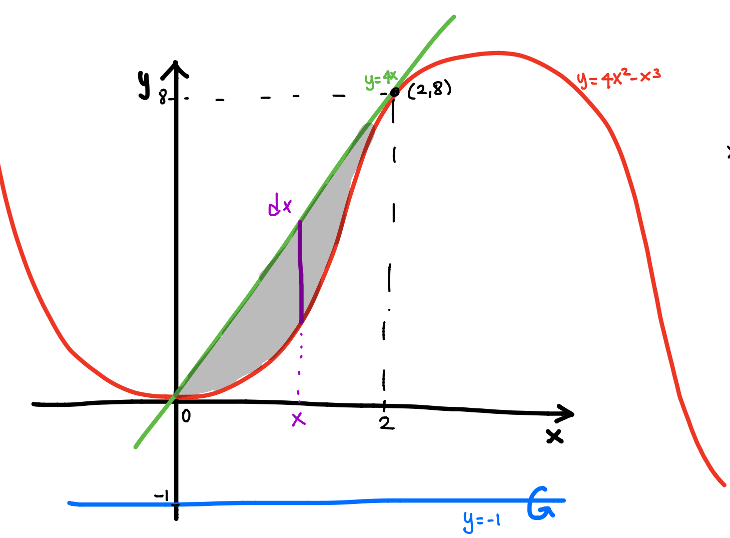
$r_{outer} = (x+2) - (-3) = x+5$

$r_{inner} = x - (-3) = x+3$

WASHER METHOD

$$V = \pi \int_0^4 (x+5)^2 - (x+3)^2 dx$$

(d) $y = 4x, \quad y = 4x^2 - x^3$ about the $y = -1$



intersection:

$4x = 4x^2 - x^3$

$x^3 - 4x^2 + 4x = 0$

$x(x^2 - 4x + 4) = 0$

$x(x-2)(x-2) = 0$

$x=0, \ x=2$

$y=0, \ y=8$

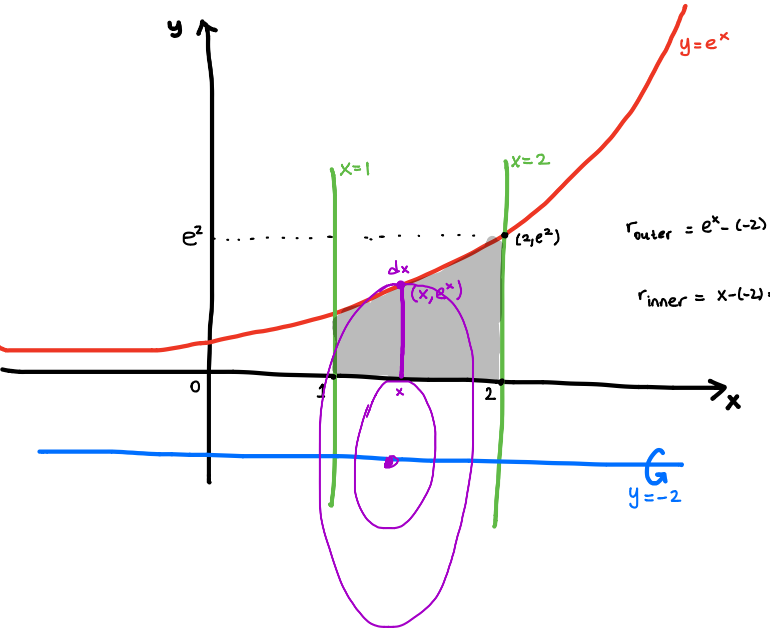
WASHER METHOD

$$\text{Volume} = \pi \int_0^2 (4x+1)^2 - (4x^2 - x^3 + 1)^2 dx$$

$r_{outer} = 4x - (-1)$
$= 4x+1$

$r_{inner} = 4x^2 - x^3 - (-1)$
$= 4x^2 - x^3 + 1$

(e) $y = e^x$, $\quad y = 0$, $\quad x = 1$, $\quad x = 2$ about $y = -2$



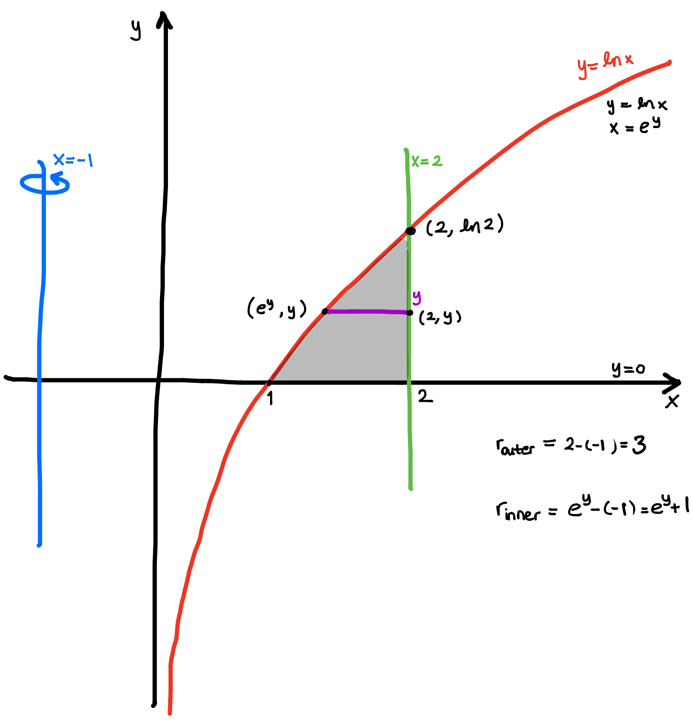
$r_{outer} = e^x - (-2) = e^x + 2$

$r_{inner} = x - (-2) = x + 2$

WASHER METHOD

$$\text{Volume} = \pi \int_1^2 (e^x+2)^2 - (x+2)^2 \, dx$$

(f) $y = \ln x$, $\quad y = 0$, $\quad x = 2$, $\quad$ about $x = -1$



$r_{outer} = 2 - (-1) = 3$

$r_{inner} = e^y - (-1) = e^y + 1$

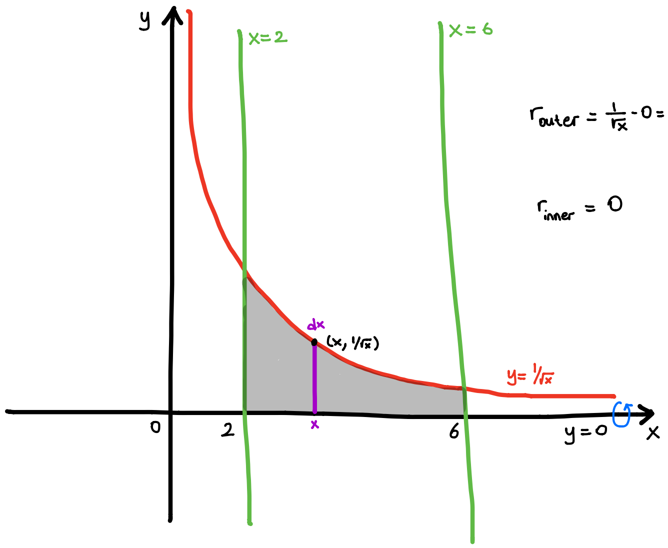
WASHER METHOD

$$\text{Volume} = \pi \int_0^{\ln 2} 3^2 - (e^y+1)^2 \, dy$$

SHELL METHOD:

$$\text{Volume} = 2\pi \int_1^2 (x+1) \ln x \, dx$$

(g) $y = \dfrac{1}{\sqrt{x}}, \quad y = 0, \quad x = 2, \quad x = 6$ about the x-axis



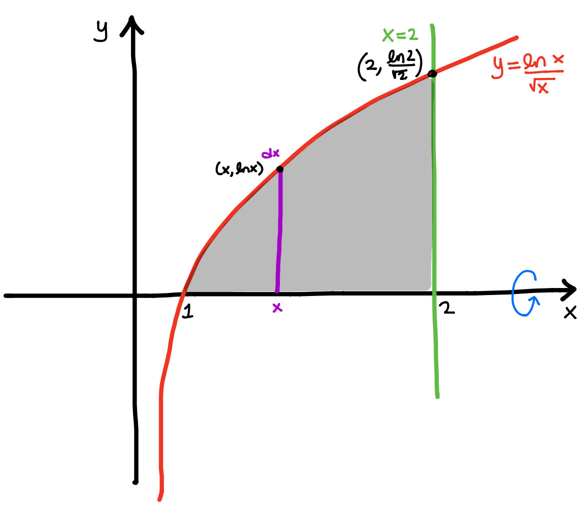
$r_{outer} = \frac{1}{\sqrt{x}} - 0 = \frac{1}{\sqrt{x}}$

$r_{inner} = 0$

WASHER METHOD

$$\text{Volume} = \pi \int_2^6 \left(\frac{1}{\sqrt{x}}\right)^2 - 0^2 \, dx$$

(h) $y = \dfrac{\ln x}{\sqrt{x}}, \quad y = 0, \quad x = 2$, about the x-axis



$r_{outer} = \frac{\ln x}{\sqrt{x}} - 0$

$= \frac{\ln x}{\sqrt{x}}$

$r_{inner} = 0$

WASHER METHOD

$$\text{Volume} = \pi \int_1^2 \left(\frac{\ln x}{\sqrt{x}}\right)^2 - 0^2 \, dx$$