SOLUTIONS Worksheet 5.5

Full Name: ________________________________ Score: ________

1. Compute each of the following integrals.

(a) $\int \sin(2x)\,dx$ $= \int \sin(u)\,\frac{du}{2} = \frac{1}{2}\int \sin(u)\,du$

$u = 2x$
$du = 2dx$
$dx = \frac{du}{2}$

$= -\frac{1}{2}\cos(u) + C$

$= -\frac{1}{2}\cos(2x) + C$

(b) $\int 8\csc(4x)\cot(4x)\,dx$ $= \int 8\csc(u)\cot(u)\,\frac{du}{4}$

$u = 4x$
$du = 4dx$
$dx = \frac{du}{4}$

$= \int 2\csc(u)\cot(u)\,du$

$= -2\csc(u) + C$

$= -2\csc(4x) + C.$

(c) $\int \frac{4}{2x-1}\,dx$ $= \int \frac{4}{u}\,\frac{du}{2} = 2\int \frac{1}{u}\,du$

$u = 2x-1$
$du = 2dx$
$dx = \frac{du}{2}$

$= 2\ln|u| + C$

$= 2\ln|2x-1| + C$

OR $\ln(2x-1)^2 + C.$

(d) $\int \frac{15}{\cos^2(5x)}\,dx$ $= \frac{1}{5}\int 15\sec^2 u\,du = 3\int \sec^2 u\,du$

$= 3\tan(5x) + C$

$u = 5x$
$du = 5dx$
$dx = \frac{du}{5}$

(e) $\int 3(5x-2)^3\,dx = \frac{1}{5}\int 3u^3\,du$

$u = 5x-2$

$du = 5dx$

$= \frac{1}{5}\cdot 3\cdot \frac{1}{4}u^4 + C$

$= \frac{3}{20}(5x-2)^4 + C$

(f) $\int \frac{(2e^x-3)^2}{e^{2x}}\,dx = \int \frac{4e^{2x}-12e^x+9}{e^{2x}}\,dx$

$= \int \frac{4e^{2x}}{e^{2x}} - \frac{12e^x}{e^{2x}} + \frac{9}{e^{2x}}\,dx$

$= \int 4 - 12e^{-x} + 9e^{-2x}\,dx$

$= 4x + 12e^{-x} - \frac{9}{2}e^{-2x} + C$

(g) $\int_0^{\frac{\sqrt{3}}{2}} \frac{3}{1+(2x)^2}\,dx = \frac{1}{2}\int_0^{\sqrt{3}} \frac{3}{1+u^2}\,du = \frac{3}{2}\tan^{-1}u \Big|_0^{\sqrt{3}}$

$u = 2x$ $\to x = \frac{\sqrt{3}}{2} \to u = 2\frac{\sqrt{3}}{2} = \sqrt{3}$; $\to x = 0 \to u = 0$

$du = 2dx$

$= \frac{3}{2}\tan^{-1}(\sqrt{3}) - \frac{3}{2}\tan^{-1}0$

$= \frac{3}{2}\cdot\frac{\pi}{3} - \frac{3}{2}\cdot 0$

$= \pi$

(h) $\int_0^5 \frac{1}{\sqrt{3x+1}}\,dx = \frac{1}{3}\int_1^{16} u^{-1/2}\,du = \frac{1}{3}\cdot 2u^{1/2}\Big|_1^{16}$

$u = 3x+1$ $\to x = 5,\ u = 16$; $\to x = 0,\ u = 1$

$du = 3dx$

$= \frac{2}{3}\sqrt{16} - \frac{2}{3}\sqrt{1}$

$= \frac{8}{3} - \frac{2}{3}$

$= 2$

(i) $\int 2x(1+x^2)^5\,dx$ $= \int u^5\,du$

$u = 1+x^2$

$du = 2x\,dx$

$= \frac{1}{6}(1+x^2)^6 + C$

(j) $\int 12x^3 e^{x^4}\,dx$ $= \int 3e^u\,du$

$u = x^4$

$du = 4x^3\,dx$

$= 3e^u + C$

$= 3e^{x^4} + C$

(k) $\int e^x \cos(e^x)\,dx$ $= \int \cos u\,du$

$u = e^x$

$du = e^x\,dx$

$= \sin u + C$

$= \sin(e^x) + C$

(l) $\int \cos^5 x \sin x\,dx$ $= -\int u^5\,du$

$u = \cos x$

$du = -\sin x\,dx$

$= -\frac{1}{6}u^6 + C$

$= -\frac{1}{6}\cos^6 x + C$

(m) $\int 24x^2 \sec(2x^3)\tan(2x^3)\,dx$ $= \int 4 \sec u \tan u \; du$

$u = 2x^3$
$du = 6x^2 dx$

$= 4 \sec u + C$

$= 4 \sec(2x^3) + C$

(n) $\int \cot x\,dx$ $= \int \frac{\cos x}{\sin x}\,dx = \int \frac{1}{u}\,du$

$u = \sin x$
$du = \cos x\,dx$

$= \ln|u| + C$

$= \ln|\sin x| + C$

(o) $\int \frac{(\ln x)^3}{x}\,dx$ $= \int \frac{u^3}{x} x\,du = \int u^3 du$

$u = \ln x$
$du = \frac{1}{x}dx$
$dx = x\,du$

$= \frac{1}{4}u^3 + C$

$= \frac{1}{4}(\ln x)^3 + C$

(p) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$ $= \int u\,du = \frac{1}{2}u^2 + C$

$u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \frac{1}{2}(\sin^{-1} x)^2 + C$

(q) $\int e^{\tan t} \sec^2 t \, dt = \int e^u du = e^u + C$

$= e^{\tan t} + C$

$u = \tan t$

$du = \sec^2 t \, dt$

(r) $\int (12x - 10)\sqrt{3x^2 - 5x} \, dx = \int 2\sqrt{u} \, du$

$= 2 \cdot \frac{2}{3} u^{3/2} + C$

$u = 3x^2 - 5x$

$du = (6x - 5) dx$

$= \frac{4}{3}(3x^2 - 5x)^{3/2} + C$

(s) $\int \frac{x^3}{1 + x^4} \, dx = \frac{1}{4} \int \frac{1}{u} du$

$= \frac{1}{4} \ln|u| + C$

$u = 1 + x^4$

$du = 4x^3 dx$

$= \frac{1}{4} \ln|1 + x^4| + C$

$= \frac{1}{4} \ln(1 + x^4) + C$

(t) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = 2 \int e^u du$

$= 2e^u + C$

$u = \sqrt{x}$

$du = \frac{1}{2} \frac{1}{\sqrt{x}} dx$

$= 2e^{\sqrt{x}} + C$