Worksheet 11.5 (Alternating series)

Full Name: ______________________________________ Score: ________

1. Test the series for convergence or divergence using the **Alternating Series Test**.

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{3n-1}{2n+1}$

$b_n = \frac{3n-1}{2n-1}$

$\lim_{n\to\infty}(b_n) = 3/2 \neq 0 \Rightarrow \lim_{n\to\infty}\left((-1)^n \frac{3n-1}{2n+1}\right)$ DNE $\neq 0$

The series diverges by the n^{th} term test.

(b) $\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{n^3+4}$

1. $b_n = \frac{n^2}{n^3+4} > 0$ for each n.

2. $f(x) = \frac{x^2}{x^3+4} \Rightarrow f'(x) = \frac{2x(x^3+4) - 3x^2(x^2)}{(x^3+4)^2} = \frac{4-x^4}{(x^3+4)^2}$

This is less than zero for $x \geq 2$ $\{b_n\}$ is decreasing for $n \geq 2$.

3. $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n^2}{n^3+4} = 0$

Therefore by AST the series converges.

(c) $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$

$\cos(n\pi) = \begin{cases} 1 & \text{if } n = \text{even} \\ -1 & \text{if } n = \text{odd} \end{cases}$

$= \sum_{n=2}^{\infty}(-1)^{n+1} \frac{1}{\sqrt{n}}$

1. $b_n = \frac{1}{\sqrt{n}} > 0$ for each n

2. $b_{n+1} = \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}} = b_n$

3. $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

By AST the series converges.