Worksheet 11.4

Full Name: ______________________________ Score: ________

1. Use **The Comparison Test** to determine if each of the following series is convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2n+3}$

$\frac{1}{n^2+2n+3} < \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (P-series. $P=2>1$). Hence,

$\sum_{n=1}^{\infty} \frac{1}{n^2+2n+3}$ converges

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3-2}$

$\frac{n^2}{n^3} < \frac{n^2}{n^3-2}$

$\frac{1}{n} < \frac{n^2}{n^3-2}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (Harmonic Series. $P=1$)

Hence $\sum_{n=1}^{\infty} \frac{n^2}{n^3-2}$ diverges.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$

$\frac{2^n}{3^n+1} < \frac{2^n}{3^n}$

$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ converges. (Geo. Series $r=\frac{2}{3}<1$)

Hence $\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$ converges also.

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{4^n-3}$

$\frac{5^n}{4^n} < \frac{5^n}{4^n-3}$

$\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^n$ diverges. (Geo. Series $r=\frac{5}{4}>1$)

Hence $\sum \frac{5^n}{4^n-3}$ diverges.

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{\frac{5}{2}}}$

$\sin^2 n \leq 1$

$\frac{\sin^2 n}{n^{5/2}} \leq \frac{1}{n^{5/2}}$

The series $\sum \frac{1}{n^{5/2}}$ Converges. (p-series, $p=\frac{5}{2}>1$)

Hence, $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{5/2}}$ converges.

2. Use **The Limit Comparison Test** to determine if each of the following series is convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n+3}$

$b_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\frac{1}{2n+3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{2n+3} \cdot \frac{n}{1} = 2 \neq 0$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ Diverges. (Harmonic series, $P=1$) so $\sum_{n=1}^{\infty} \frac{1}{2n+3}$ Diverges as well.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n\sqrt{n}}$

$b_n = \frac{n}{n\sqrt{n}} = \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{n-1}{n\sqrt{n}} \cdot \frac{\sqrt{n}}{1} = 1 \neq 0$$

The series $\sum \frac{1}{\sqrt{n}}$ is div. (p series with $p=\frac{1}{2}$)

Hence the given series is div. by lim comp. test

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2n-1}$

$b_n = \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{2n-1} \cdot \frac{\sqrt{n}}{1} = \frac{1}{2} \neq 0$$

$\sum \frac{1}{\sqrt{n}}$ diverges. Hence the given series diverges.

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^4+3n^2-2n+4}{-5n^6+n^4-2n^3+n-15}$

Since some (or) all of the terms are negative we cannot use the limit comparison test directly. However, we can apply the limit comparison test in a modified version as follows:

$$\sum_{n=1}^{\infty} \frac{2n^4+3n^2-2n+4}{-(5n^6-n^4+2n^3-n+15)} = -\sum_{n=1}^{\infty} \frac{2n^4+3n^2-2n+4}{5n^6-n^4+2n^3-n+15}$$

Apply the limit comparison test to the new series $\displaystyle\sum_{n=1}^{\infty} \frac{2n^4+3n^2-2n+4}{5n^6-n^4+2n^3-n+15}$, $b_n = \frac{1}{n^2}$

$\displaystyle\lim_{n\to\infty} \frac{2n^4+3n^2-2n+4}{5n^6-n^4+2n^3-n+15} \cdot \frac{n^2}{1} = \frac{2}{5} \neq 0$; $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, hence $\displaystyle\sum_{n=1}^{\infty} \frac{2n^4+3n^2-2n+4}{5n^6-n^4+2n^3-n+15}$ converges.

Then, $\displaystyle\sum_{n=1}^{\infty} \frac{2n^4+3n^2-2n+4}{-5n^6+n^4-2n^3+n-15}$ converges.

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2+2n^3+3n-2}{-n+n^5+25}$

$b_n = \frac{1}{n^2}$

$\displaystyle\lim_{n\to\infty} \frac{5n^2+2n^3+3n-2}{-n+n^5+25} \cdot \frac{n^2}{1} = 2$, $\sum \frac{1}{n^2}$ converges. Hence $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2+2n^3+3n-2}{-n+n^5+25}$ converges.

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^{\frac{3}{2}}+3n+5}{-5n^2+n+3} = -\sum_{n=1}^{\infty} \frac{2n^{3/2}+3n+5}{5n^2-n-3}$

limit comparison. $b_n = \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{2n^{3/2}+3n+5}{5n^2-n-3} \cdot \frac{\sqrt{n}}{1} = \frac{2}{5}$$

$\sum \frac{1}{\sqrt{n}}$ diverges. Hence $\displaystyle\sum_{n=1}^{\infty} \frac{2n^{3/2}+3n+5}{-5n^2+n+3}$ diverges.

(g) $\sum_{n=1}^{\infty} \frac{3^n - 1}{4^n - 1}$

$b_n = \frac{3^n}{4^n}$ $\qquad \left| \frac{3^n - 1}{4^n - 1} \cdot \frac{4^n}{3^n} \right| \xrightarrow{n \to \infty} 1$

$\sum_{n=1}^{\infty} \frac{3^n}{4^n}$ converges (geometric series with r=3/4). Hence by the limit comparison test the series $\sum_{n=1}^{\infty} \frac{3^n - 1}{4^n - 1}$ converges as well.

(h) $\sum_{n=1}^{\infty} \frac{5n^2 + 3^n + 7}{2n^4 + 2^n - 1}$

$b_n = \left(\frac{3}{2}\right)^n$ $\qquad \lim_{n \to \infty} \frac{a_n}{b_n} = 1$

$\sum \left(\frac{3}{2}\right)^n$ div. Hence $\sum \frac{5n^2 + 3^n + 7}{2n^4 + 2^n - 1}$ diverges as well.

(i) $\sum_{n=1}^{\infty} \frac{2n^2 + 5}{\sqrt{9n^6 + n^2 + 8}}$

$b_n = \frac{1}{n}$ $\qquad \left(\frac{n^2}{\sqrt{n^6}}\right)$

$\lim_{n \to \infty} \frac{2n^2 + 5}{\sqrt{9n^6 + n^2 + 8}} \cdot \frac{n}{1} = \lim_{n \to \infty} \frac{2n^3}{\sqrt{9n^6}} = \frac{2}{3}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges hence $\sum_{n=1}^{\infty} \frac{2n^2 + 5}{\sqrt{9n^6 + n^2 + 8}}$ diverges.