Worksheet 11.6 and 11.8

Full Name: ______________________________ Score: ________

1. Determine whether each series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

absolutely conv.

$$\sum_{n=1}^{\infty} \left| \frac{\sin n}{n^2} \right|$$

$\left|\frac{\sin n}{n^2}\right| \leq \frac{1}{n^2}$, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ is conv. (p.series)

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[3]{n}}$

$\displaystyle\sum_{n=1}^{\infty} \left| (-1)^n \frac{1}{\sqrt[3]{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} \rightarrow$ Div. by p-series test

$\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[3]{n}}$ conv. by A.S.T.

Hence it is conditionally convergent.

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^4}$

$\displaystyle\sum_{n=1}^{\infty} \left| (-1)^n \frac{1}{n^4} \right| = \sum_{n=1}^{\infty} \frac{1}{n^4} \rightarrow$ conv. by p-series

Hence $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ is absolutely convergent

(d) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{2n+3}$

$\sum_{n=1}^{\infty}\left|(-1)^n \frac{1}{2n+3}\right| = \sum_{n=1}^{\infty} \frac{1}{2n+3}$ Divergent. (lim. comp. to $\sum_{n=1}^{\infty} \frac{1}{n}$)

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{2n+3} \rightarrow$ Conv. by A.S.T.

Conditionally conv.

(e) $\sum_{n=1}^{\infty}(-1)^n \frac{n-1}{n\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{n-1}{n\sqrt{n}}$ div. by lim. comp. (let $b_n = \frac{1}{\sqrt{n}}$)

$\sum_{n=1}^{\infty} (-1)^n \frac{n-1}{n\sqrt{n}}$ conv. by A.S.T.

Conditionally convergent.

(f) $\sum_{n=1}^{\infty} \cos(n\pi) \frac{\sqrt{n}}{2n-1}$

$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2n-1}$ Div. by lim. comp. (let $b_n = \frac{1}{\sqrt{n}}$)

$\sum_{n=1}^{\infty} \cos(n\pi) \frac{\sqrt{n}}{2n-1}$ is conv. by A.S.T

Conditionally conv.

(g) $\sum_{n=1}^{\infty}(-1)^n \frac{2n^4+3n^2-2n+4}{-5n^6+n^4-2n^3+n-15} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n^4+3n^2-2n+4}{5n^6-n^4+2n^3-n+15}$

$\sum_{n=1}^{\infty} \left| \frac{2n^4+3n^2-2n+4}{5n^6-n^4+2n^3-n+15} \right|$ is conv. by lim. comp. $\left(b_n = \frac{1}{n^2}\right)$

abs. conv.

(h) $\sum_{n=1}^{\infty}(-1)^n \frac{5n^2+2n^3+3n-2}{-n+n^5+25}$

$\sum_{n=1}^{\infty} \left| \frac{5n^2+2n^3+3n-2}{-n+n^5+25} \right|$ is conv. by lim. comp. $\left(b_n = \frac{1}{n^2}\right)$

abs. conv.

(i) $\sum_{n=1}^{\infty}(-1)^n \frac{2n^{\frac{3}{2}}+3n+5}{-5n^2+n+3} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n^{3/2}+3n+5}{5n^2-n-3}$

$\sum_{n=1}^{\infty} \left| \frac{2n^{3/2}+3n+5}{5n^2-n-3} \right|$ is div. by lim. comp. $\left(b_n = \frac{1}{\sqrt{n}}\right)$

$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n^{3/2}+3n+5}{5n^2-n-3}$ conv. by A.S.T.

Cond. conv.

(j) $\sum_{n=1}^{\infty}(-1)^n \frac{3^n-1}{4^n-1}$

$\sum_{n=1}^{\infty}\left|\frac{3^n-1}{4^n-1}\right|$ → conv. by lim. comparison test $\left(b_n = \left(\frac{3}{4}\right)^n\right)$

Absolutely convergent.

(k) $\sum_{n=1}^{\infty}(-1)^n \frac{5n^2+3^n+7}{2n^4+2^n-1}$

$\sum_{n=1}^{\infty}\left|\frac{5n^2+3^n+7}{2n^4+2^n-1}\right|$ → divergent by limit comparison test $\left(b_n = \frac{3^n}{2^n}\right)$

$\sum_{n=1}^{\infty}(-1)^n \frac{5n^2+3^n+7}{2n^4+2^n-1}$ → divergent by divergence test. $\left(\lim_{n\to\infty} (-1)^n \frac{5n^2+3^n+7}{2n^4+2^n-1} = \text{DNE}\right)$

The series is Divergent

(l) $\sum_{n=1}^{\infty}(-1)^n \frac{2n^2+5}{\sqrt{9n^6+n^2+8}}$

$\sum_{n=1}^{\infty}\frac{2n^2+5}{\sqrt{9n^6+n^2+8}}$ div. by lim. comp. test $\left(b_n = \frac{1}{n}\right)$

$\sum_{n=1}^{\infty}(-1)^n \frac{2n^2+5}{\sqrt{9n^6+n^2+8}}$ convergent by A.S.T.

The series is conditionally convergent.

2. Use the **Ratio Test** or the **Root Test** to determine whether each series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n2^n}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{(n+1)^2+1}{(n+1)2^{n+1}} \cdot \frac{n2^n}{n^2+1} \right|$$

$$\lim_{n\to\infty} \left| \frac{n((n+1)^2+1)}{(n^2+1)(n+1)2} \right| = \lim_{n\to\infty} \frac{n^3}{n^3 2} = \frac{1}{2} < 1$$, The series converges.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{(-3)^{n+1}}{(n+1)n!} \cdot \frac{n!}{(-3)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{-3}{n+1} \right| = 0 < 1$$

The series converges.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)3^n}{n^3 2^n}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{(n+1+1)3^{n+1}}{(n+1)^3 \cdot 2^{n+1}} \cdot \frac{n^3 \cdot 2^n}{(n+1)3^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+2)n^3 \cdot 3}{(n+1)^3 \cdot (n+1) \cdot 2} \right|$$

$$= \frac{3}{2} > 1$$, The series diverges.

(d) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{2n^2}{3n^2+n+10}\right)^n$

Root Test

$$\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{2n^2}{3n^2+n+10}\right)^n\right|} = \frac{2}{3} < 1$$, The series converges.

(e) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3n^4+n^2+5}{2n^4-n^3+n}\right)^n$

Root Test

$$\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{3n^4+n^2+5}{2n^4-n^3+n}\right)^n\right|} = \frac{3}{2} > 1$$, The series diverges.

(f) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{n^3+1}{2n^2+5}\right)^n$

Root Test

$$\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{n^3+1}{2n^2+5}\right)^n\right|} = \infty$$ The series diverges.

(g) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{4n^2}{2n^3+17}\right)^n$

Root Test

$$\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{4n^2}{2n^3+17}\right)^n\right|} = 0 < 1$$, The series converges.

3. Find the open interval of convergence for each power series. Show your work.

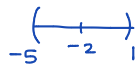
(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^2 3^n}$

center: $a = -2$

Radius: $R = \lim_{n\to\infty} \left| \frac{1}{n^2 \cdot 3^n} \cdot \frac{(n+1)^2 \cdot 3^{n+1}}{1} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)^2}{n^2} \cdot 3 \right|$

$= 3$



open int. conv
$(-5, 1)$

End Points:

$x=-5 \rightarrow \sum \frac{(-5+2)^n}{n^2 3^n} = \sum_{n=1}^{\infty} \frac{(-3)^n}{n^2 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \rightarrow$ conv A.S.T.

$x=1 \quad \sum_{n=1}^{\infty} \frac{3^n}{n^2 3^n} = \sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow$ Conv. p-series

interval of conv: $[-5, 1]$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{2n^3+10}$

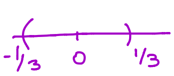
center: $a = 0$

Radius: $R = \lim_{n\to\infty} \left| \frac{3^n}{2n^3+10} \cdot \frac{2(n+1)^3+10}{3^{n+1}} \right|$

$= \lim_{n\to\infty} \left| \frac{2(n+1)^3+10}{(2n^3+10)\cdot 3} \right|$

$= \lim_{n\to\infty} \left| \frac{2n^3}{2n^3 \cdot 3} \right| = \frac{1}{3}$

$R = \frac{1}{3}$



Open interval: $(-1/3, 1/3)$

End Points:

$x = -1/3$: $\sum_{n=0}^{\infty} \frac{3^n \cdot (-\frac{1}{3})^n}{2n^3+10} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n^3+10}$

Conv. by A.S.T

$x = 1/3$: $\sum_{n=0}^{\infty} \frac{3^n (\frac{1}{3})^n}{2n^3+10} = \sum_{n=0}^{\infty} \frac{1}{2n^3+10}$

Conv. by lim. comp. test. ($b_n = \frac{1}{n^3}$ – p-series)

So the interval of convergence is:

$[-1/3, 1/3]$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n (x-3)^n}{n 10^n}$

Center: $a = 3$

$$R = \lim_{n\to\infty} \left| \frac{5^n}{n 10^n} \cdot \frac{(n+1) 10^{n+1}}{5^{n+1}} \right|$$

$$= \lim_{n\to\infty} \frac{(n+1) \cdot 10}{n \cdot 5}$$

$$= 2$$

$R = 2$

Open interval of conv:

$(1, 5)$

End points:

$x = 1$: $\displaystyle\sum_{n=1}^{\infty} \frac{5^n(-2)^n}{n 10^n} = \sum_{n=1}^{\infty} \frac{(-10)^n}{n \cdot 10^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ conv. by A.S.T.

$x = 5$: $\displaystyle\sum_{n=1}^{\infty} \frac{5^n \cdot 2^n}{n 10^n} = \sum_{n=1}^{\infty} \frac{10^n}{n 10^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ div. by p-series test

So the interval of convergence is:

$[1, 5)$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-4)^n x^n}{n!}$

center: $a = 0$

$$R = \lim_{n\to\infty} \left| \frac{(-4)^n}{n!} \cdot \frac{(n+1) \cdot n!}{(-4)^{n+1}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)}{-4} \right|$$

$$= \infty$$

Radius of conv. is ∞. So the interval of conv is $(-\infty, \infty)$