

# SERIES CONVERGENCE/DIVERGENCE FLOW CHART

## TEST FOR DIVERGENCE

Does  $\lim_{n \rightarrow \infty} a_n = 0$ ? NO  $\rightarrow \sum a_n$  Diverges

YES

## *p*-SERIES

Does  $a_n = 1/n^p, n \geq 1$ ? YES  $\rightarrow$  Is  $p > 1$ ? YES  $\rightarrow \sum a_n$  Converges  
NO  $\rightarrow \sum a_n$  Diverges

NO

## GEOMETRIC SERIES

Does  $a_n = ar^{n-1}, n \geq 1$ ? YES  $\rightarrow$  Is  $|r| < 1$ ? YES  $\rightarrow \sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$   
NO  $\rightarrow \sum a_n$  Diverges

NO

## ALTERNATING SERIES

Does  $a_n = (-1)^n b_n$  or  $a_n = (-1)^{n-1} b_n, b_n \geq 0$ ? YES  $\rightarrow$  Is  $b_{n+1} \leq b_n$  &  $\lim_{n \rightarrow \infty} b_n = 0$ ? YES  $\rightarrow \sum a_n$  Converges

NO

## TELESCOPING SERIES

Do subsequent terms cancel out previous terms in the sum? May have to use partial fractions, properties of logarithms, etc. to put into appropriate form. YES  $\rightarrow$  Does  $\lim_{n \rightarrow \infty} s_n = s$  YES  $\rightarrow \sum a_n = s$   
NO  $\rightarrow \sum a_n$  Diverges

NO

## TAYLOR SERIES

Does  $a_n = \frac{f^{(n)}(a)}{n!} (x-a)^n$ ? YES  $\rightarrow$  Is  $x$  in interval of convergence? YES  $\rightarrow \sum_{n=0}^{\infty} a_n = f(x)$   
NO  $\rightarrow \sum a_n$  Diverges

NO

Try one or more of the following tests:

## COMPARISON TEST

Pick  $\{b_n\}$ . Does  $\sum b_n$  converge? YES  $\rightarrow$  Is  $0 \leq a_n \leq b_n$ ? YES  $\rightarrow \sum a_n$  Converges  
NO  $\rightarrow$  Is  $0 \leq b_n \leq a_n$ ? YES  $\rightarrow \sum a_n$  Diverges

## LIMIT COMPARISON TEST

Pick  $\{b_n\}$ . Does  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$   $c$  finite &  $a_n, b_n > 0$ ? YES  $\rightarrow$  Does  $\sum_{n=1}^{\infty} b_n$  converge? YES  $\rightarrow \sum a_n$  Converges  
NO  $\rightarrow \sum a_n$  Diverges

## INTEGRAL TEST

Does  $a_n = f(n)$ ,  $f(x)$  is continuous, positive & decreasing on  $[a, \infty)$ ? YES  $\rightarrow$  Does  $\int_a^{\infty} f(x) dx$  converge? YES  $\rightarrow \sum_{n=a}^{\infty} a_n$  Converges  
NO  $\rightarrow \sum a_n$  Diverges

## RATIO TEST

Is  $\lim_{n \rightarrow \infty} |a_{n+1}/a_n| \neq 1$ ? YES  $\rightarrow$  Is  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$ ? YES  $\rightarrow \sum a_n$  Abs. Conv.  
NO  $\rightarrow \sum a_n$  Diverges

## ROOT TEST

Is  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \neq 1$ ? YES  $\rightarrow$  Is  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$ ? YES  $\rightarrow \sum a_n$  Abs. Conv.  
NO  $\rightarrow \sum a_n$  Diverges

1. 
$$\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$$

2. 
$$\sum_{n=1}^{\infty} \frac{n - 1}{n^2 + n}$$

3. 
$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$

4. 
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n - 1}{n^2 + n}$$

5. 
$$\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$$

6. 
$$\sum_{n=1}^{\infty} \left( \frac{3n}{1 + 8n} \right)^n$$

7. 
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$$

8. 
$$\sum_{k=1}^{\infty} \frac{2^k k!}{(k + 2)!}$$

9. 
$$\sum_{k=1}^{\infty} k^2 e^{-k}$$

10. 
$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

11. 
$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln(n)}$$

12. 
$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 25}$$

13. 
$$\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

14. 
$$\sum_{n=1}^{\infty} \sin(n)$$

15. 
$$\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n + 2)}$$

16. 
$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1}$$

17. 
$$\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$$

18. 
$$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n} - 1}$$

19. 
$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$$

20. 
$$\sum_{k=1}^{\infty} \frac{k + 5}{5^k}$$

21. 
$$\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$$

22. 
$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5}$$

23. 
$$\sum_{n=1}^{\infty} \tan(1/n)$$

24. 
$$\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$$

25. 
$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

26. 
$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$$

27. 
$$\sum_{k=1}^{\infty} \frac{k \ln(k)}{(k + 1)^3}$$

28. 
$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

29. 
$$\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n\sqrt{n}}$$

30. 
$$\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j + 5}$$

31. 
$$\sum_{k=1}^{\infty} \frac{5^k}{3^k + 4^k}$$

32. 
$$\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$$

33. 
$$\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$$

34. 
$$\sum_{n=1}^{\infty} \frac{1}{n + n \cos^2(n)}$$

35. 
$$\sum_{n=1}^{\infty} \left( \frac{n}{n + 1} \right)^{n^2}$$

36. 
$$\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$$

37. 
$$\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$$

38. 
$$\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)$$