16 1. Find the derivatives of the following functions. You do not need to simplify your answer.

(a) $f(x) = \left(\cos^{-1} x + x^2\right)^2$

(b) $f(x) = \dfrac{\tan x}{1 - \sec x}$

(c) $f(x) = e^{x^3 \ln x}$

(d) $f(x) = 7^{(4x^3+x)}$

Solution: `ww2.coastal.edu/rdahal/math160/ap2`

8 2. Find $\dfrac{dy}{dx} = y'$ by implicit differentiation. $xy - \cos(xy) = 1$

$$\begin{aligned} \text{Here, we have} \qquad xy - \cos(xy) &= 1 \\ \text{(diff. w/r to } x\text{, we get,)} \qquad (1y + y'x) + \sin(xy)(1y + y'x) &= 0 \\ y + y'x + y\sin(xy) + xy'\sin(xy) &= 0 \\ y'(x + x\sin(xy)) &= -y - y\sin(xy) \\ y' &= \frac{-y - y\sin(xy)}{x + x\sin(xy)}. \end{aligned}$$

13 3. A potato is launched vertically upward with an initial velocity of 160 ft/sec from the ground. The distance s in feet that the potato travels from the ground after t seconds is given by $s(t) = -16t^2 + 160t$. Answer the following questions with correct units.

(a) Determine the velocity of the potato after 4 sec and 6 sec.

Here $v(t) = s'(t) = -32t + 160$. So $v(4) = 32$ ft/sec, and $v(6) = -32$ ft/sec.

(b) Determine when the potato reaches its maximum height.

Max height $\implies v(t) = 0 \implies -32t + 160 = 0 \implies t = 5$ seconds.

(c) Determine the velocity of the potato upon hitting the ground.

Since it takes 5 seconds to reach max. height we know that it takes 10 seconds to get back to the ground. So $v(10) = -32 * 10 + 160 = -160$ ft/sec.

9 4. Determine the absolute maximum and minimum of $f$ on the given interval.

$$f(x) = x^3 - 6x^2 + 9x + 2, \qquad [-2, 2]$$

Solution: #3(b) here: `http://ww2.coastal.edu/rdahal/math160/extreme_values/`

[8] 5. Use a linear approximation to estimate $(1.01)^3$.

Solution: #4 here: `http://ww2.coastal.edu/rdahal/math160/linearization/`

[10] 6. A 10 feet ladder is leaning against a vertical wall. If the top of the ladder slides down the wall at a rate of $\sqrt{2}$ feet per second, how fast is the bottom of the ladder moving along the ground when the bottom of the ladder is 5 feet from the wall? (Answer = 2 ft/sec)

Solution: Similar to #4 here `http://ww2.coastal.edu/rdahal/math160/related_rates/`

[7] 7. Sketch the graph of a function $f$ that satisfies all of the given conditions.

1. $f(-2) = 0, \quad f'(-2) = 0,$ and $f(0) = 3,$
2. $f'(x) < 0$ on the interval $(-\infty, -2)$, and $f'(x) > 0$ on $(-2, 0)$
3. $f''(x) > 0$ on the interval $(-\infty, -1)$, and $f''(x) < 0$ on $(-1, 1)$
4. The graph has $y$-axis symmetry.

Solution: #B here: `http://ww2.coastal.edu/rdahal/math160/shape_of_graph/`

8. Consider the function $f(x) = x^3 + 3x^2 - 24x + 10$. Answer the following with your calculus work.

Solution: #B. here: `http://ww2.coastal.edu/rdahal/math160/shape_of_graph/`

[4] (a) Find the intervals on which $f$ is increasing or decreasing.

[2] (b) Find the local maximum and minimum **values** of $f$. [(Answer: local max = 90, local min = -18)].

[4] (c) Find the intervals on which $f$ is concave up or concave down.

[2] (d) Find the inflection point(s) of $f$. [Answer: (-1, 36)].

[8] 9. The Mean Value Theorem guarantees the existence of a special number $c$ in the interval $(1, e)$ for the function $f(x) = \ln(x)$. Find the number $c$. (Answer = 1.72)

Find the solution here: `https://ximera.osu.edu/andrewcalc/calcBook/calcBook/mvt/mvt`

More questions on the next page.

Circle the correct answer. You do not need to show your work. (No partial credit will be given.)

3 10. If $f(x) = \log_5(2x)$. Find the value of $f'(3)$.

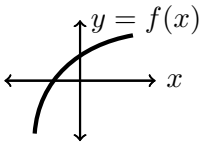
(a) $\frac{1}{3}$ (b) $\frac{1}{3 \ln 5}$ (c) $\frac{1}{6 \ln 5}$ **(d) $\frac{1}{3 \ln 5}$** (e) 1

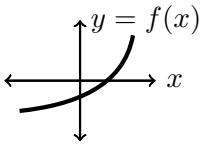
3 11. The position of a bird flying along a straight line in $t$ seconds is given by $s(t) = -2t^2 + t$ meters. What is the acceleration (in $meters/sec^2$) after 1 second?

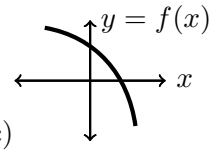
(a) 2 **(b) $-4$** (c) 9 (d) 18 (e) 11

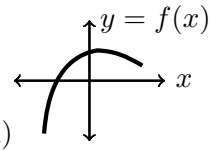
3 12. If $y = f(x)$ is a function such that $f' > 0$ for all $x$ and $f'' > 0$ for all $x$, which of the following could be part of the graph of $y = f(x)$?

**The two conditions imply increasing and concave up so the correct choice is (b).**

(a) 


(b) 


(c) 


(d) 


(e)