[8] 1. (a) Find $\frac{dy}{dx} = y'$ by implicit differentiation. $2y + \sin x - 2 = x^4 - xy$.

$$\begin{aligned} \text{Here} \quad 2y + \sin x - 2 &= x^4 - xy \\ \text{diff. w/r to } x\text{, we get} \quad 2y' + \cos x &= 4x^3 - (y + y'x) \\ 2y' + y'x &= 4x^3 - y - \cos x \\ y'(2+x) &= 4x^3 - y - \cos x \\ y' &= \frac{4x^3 - y - \cos x}{2+x}. \end{aligned}$$

[6] (b) Find an **equation of the tangent line** to the curve

$$2y + \sin x - 2 = x^4 - xy \qquad \text{at the point } (0,1),$$

in the slope-intercept ($y = mx + b$) form. (Note that it is the same curve in part (a) above.)

Slope, $m = \frac{0 - 1 - \cos 0}{2+0} = \frac{-2}{2} = -1$. Point $(0,1)$ means the y-intercept, $b = 1$.

Thus an equation of the tangent line is $y = -x + 1$.

[12] 2. Find the derivatives of the following functions. You do not need to simplify your answer.

(a) $f(x) = e^{5x} + x^5 + 5^x$

(b) $f(x) = \ln(x \tan x)$

(c) $f(x) = \left(\frac{x^3 + 2x}{5x^2 - 3}\right)^7$

Solution: `ww2.coastal.edu/rdahal/math160/ap2`

[6] 3. An object is launched vertically up on Planet X. The position, $s(t)$ (in feet), of the object after t seconds is given by

$$s(t) = 42t - 6t^2$$

What is the maximum height reached by the object?

We have $v(t) = s'(t) = 42 - 12t$. Max. height $\implies v(t) = 0$. That is, $42 - 12t = 0 \implies t = 3.5$ seconds.

Now the max. height is $s(3.5) = 42 * 3.5 - 6 * (3.5)^2 = 73.5$ feet.

[10] 4. Find the linearization $L(x)$ of the function $f(x) = \sqrt[3]{x}$ at the number 27, and use it to approximate the value of $\sqrt[3]{28}$.

Here $a = 27$. We have $f(a) = \sqrt[3]{27} = 3$. Next $f'(x) = \frac{1}{3x^{2/3}}$, so we get $f'(27) = \frac{1}{27}$.

Now using the formula, $f(x) \approx L(x) = f(a) + f'(a)(x - a)$, we get $L(x) = 3 + \frac{1}{27}(x - 27)$.

Then $\sqrt[3]{28} \approx 3 + \frac{1}{27}(28 - 27) = 3 + \frac{1}{27} * 1 = 3 + \frac{1}{27} = \frac{82}{27}$.

5. Consider the function $f(x) = 5x^{\frac{1}{3}} - x^{\frac{5}{3}}$. Answer the following using **calculus**.

[6] (a) Find the intervals on which f is increasing or decreasing.

[2] (b) Find the $x-$values where f attains its local maximum and minimum values.

Solution: `ww2.coastal.edu/rdahal/math160/ap3`

[8] 6. Note that the function $f(x) = 3x^2 + 2x + 5$ is continuous on $[-1, 2]$ and differentiable on $(-1, 2)$ being a polynomial. Find all number(s) **c** in $(-1, 2)$ satisfying the conclusion of the Mean Value Theorem for the function f.

Solution: #4 here: `http://ww2.coastal.edu/rdahal/math160/mvt/`

[10] 7. A girl flies a kite at a height of 600 ft, the wind carrying the kite horizontally away from her at a rate of 12 ft/sec. How fast must she let out the string when the kite is 1000 ft away from her? Assume the string is taut so that it forms a straight line. Answer: $\frac{48}{5}$ ft/sec.

Solution: Similar to #8 here: `http://ww2.coastal.edu/rdahal/math160/related_rates/`

[10] 8. Find the absolute maximum and absolute minimum **values** of the function $f(x) = x^3 - 3x^2 + 60$ in the interval $[-3, 1]$.

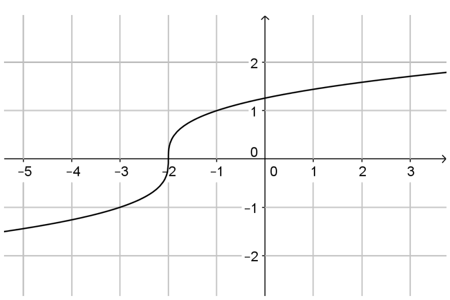
Here $f'(x) = 3x^2 - 6x$. Set $f'(x) = 0 \implies 3x^2 - 6x = 0 \implies 3x(x-2) = 0 \implies x = 0, 2$, critical numbers of f but 2 is not in the domain. So we consider $x = 0$ only as a critical number.

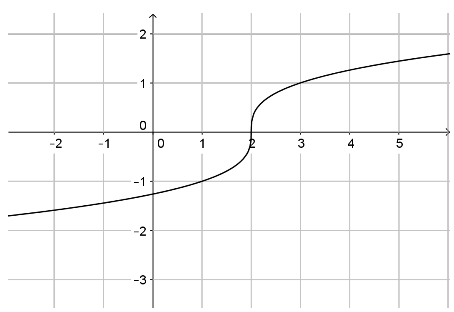
Now $f(0) = 60$, $f(-3) = 6$, $f(1) = 58$. Thus abs. max. value is 60, and abs. min. value is 6.

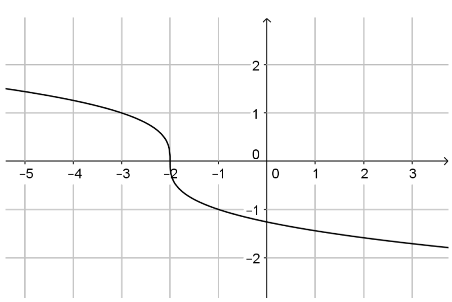
[4] 9. Choose a graph of the function f that is continuous on $(-\infty, \infty)$ and has the following properties.

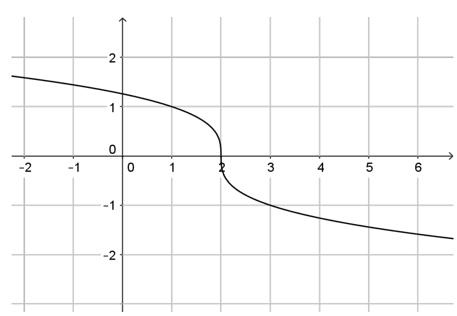
$$f'(x) < 0 \text{ and } f''(x) < 0 \text{ on } (-\infty, 2); \quad f'(x) < 0 \text{ and } f''(x) > 0 \text{ on } (2, \infty)$$

Solution: The first two conditions imply f is decreasing and concave downward on $(-\infty, 2)$ which is enough to choose **the last graph** as the answer.









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6 10. Find the **slope** of the tangent line to the curve

$$\tan(xy) = y \qquad \text{at the point} \qquad \left(\frac{\pi}{4}, 1\right).$$

Solution: `ww2.coastal.edu/rdahal/math160/ap2`

Circle the correct answer. You do not need to show your work. (No partial credit will be given.)

3 11. If $f(x) = \cos^2 x$, its derivative, $f'(x)$, is given by

(a) $\frac{\cos^3 x}{3}$ (b) $-\frac{\sin^3 x}{3}$ (c) $-\sin^2 x$ (d) $-2\cos x \sin x$ (e) $2\sin x \cos x$

3 12. The derivative of $y(x) = 3\log_3(x)$ is given by

(a) $\frac{3}{x}$ (b) $\frac{1}{x}$ (c) $\frac{1}{x\ln 3}$ (d) $\frac{3}{x\ln 3}$ (e) $\frac{3}{\ln 3}$

Use the table for the following two questions.

x	f	f'	g	g'
0	2	-2	2	-1
1	3	10	2	10
2	2	-3	-2	4

3 13. If $H(x) = f(g(x)) + 4$, find $H'(0)$.

(a) 2 (b) 4 (c) 5 (d) 3 (e) -2

3 14. If $J(x) = \tan^{-1}(g(x))$, find $J'(1)$.

(a) 0 (b) 1 (c) 10 (d) 2 (e) $\frac{1}{10}$