16 1. Find the derivatives of the following functions. You do not need to simplify your answer.

(a) $f(x) = \left(1 + \tan^{-1} x\right)^3$

(b) $f(x) = \sqrt{\dfrac{x^2 - 5}{3x + 2}}$

(c) $f(x) = \sin^2(3x)$

(d) $f(x) = \ln(4x^3 + x)$

Solution: `ww2.coastal.edu/rdahal/math160/ap2`

8 2. Find $\dfrac{dy}{dx} = y'$ by implicit differentiation. $\cos(x^2y^3) = 5x^2 - 2$

(Given) $\cos(x^2y^3) = 5x^2 - 2$

(diff. w/r to x, we get,) $-\sin(x^2y^3)[2xy^3 + 3x^2y^2y'] = 10x$

$$2xy^3 + 3x^2y^2y' = \frac{10x}{-\sin(x^2y^3)}$$

$$y' = \frac{1}{3x^2y^2}\left[-\frac{10x}{\sin(x^2y^3)} - 2xy^3\right].$$

8 3. Find an **equation of the tangent line** to the curve $y = \sin(\tan x)$ at $x = 0$.

Solution: `ww2.coastal.edu/rdahal/math160/ap2`

4. A ball is thrown upward with 80 ft/sec of velocity from a 64-foot-tall building. After t seconds, its height above the ground is given by $s(t) = -16t^2 + 80t + 64$.

2 (a) What is the **velocity** of the ball after 2 sec?

We have $v(t) = s'(t) = -32t + 80$. So $v(2) = -64 + 80 = 16$ ft/sec.

4 (b) What is the **maximum height** reached by the ball?

At max height, we have $v(t) = 0 \implies -32t + 80 = 0 \implies t = 2.5$.

Then max. height, $s(2.5) = -16 * (2.5)^2 + 80 * 2.5 + 64 = 164$ feet.

6 5. Determine the absolute maximum and minimum of f on the given interval.

$$f(x) = x^2 - 4, \qquad [-1, 4]$$

Here $f'(x) = 2x$. Set $f'(x) = 0 \implies 2x = 0 \implies x = 0$, a critical number in the domain.

Now $f(0) = -4$, $f(-1) = -3$, $f(4) = 12$. Thus abs. max. value is 12, and abs. min. value is -4.

[6] 6. Find the linear approximation $L(x)$ to $f(x) = \sqrt{x}$ at $x = 9$ and use the approximation to estimate $\sqrt{9.1}$.

Here $a = 9$. We have $f(a) = \sqrt{9} = 3$. Next $f'(x) = \dfrac{1}{2\sqrt{x}}$, so we get $f'(9) = \dfrac{1}{6}$.

Now using the formula, $f(x) \approx L(x) = f(a) + f'(a)(x - a)$, we get $L(x) = 3 + \dfrac{1}{6}(x - 9)$.

Then $\sqrt{9.1} \approx 3 + \dfrac{1}{6}(9.1 - 9) = 3 + \dfrac{1}{6} * 0.1 = 3 + \dfrac{1}{60} = \dfrac{181}{60}$.

[6] 7. Given the function $f(x) = x^3$ on the interval $[0, 2]$, determine if the Mean Value Theorem applies. If so, find all points c which satisfy the conclusion of the theorem. If not, explain why the theorem doesn't apply.

Since the function $f(x) = x^3$ is continuous on $[0, 2]$, and differentiable on $(0, 2)$ being a polynomial the MVT applies.

Now $f'(x) = 3x^2$, and $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2) - f(0)}{2 - 0} = \dfrac{8 - 0}{2} = 4$.

Setting $3x^2 = 4 \implies x = \pm\dfrac{2}{\sqrt{3}}$. But $-\dfrac{2}{\sqrt{3}}$ is outside of the given interval $(0, 2)$, so $c = \dfrac{2}{\sqrt{3}}$.

[9] 8. A balloon is rising at a constant speed of 5 ft per sec. A boy is cycling along a straight road at a speed of 15 ft per sec. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 sec later?

Solution: #9 here: `http://ww2.coastal.edu/rdahal/math160/related_rates/`

[7] 9. Sketch the graph of a function f that satisfies all of the given conditions.

1. $f(0) = 4, \quad f(2) = -2,$ and $f(4) = 1$,
2. $f'(0) = f'(2) = f'(4) = 0$,
3. $f'(x) > 0$ if $x < 0$ or $2 < x < 4$,
4. $f'(x) < 0$ if $0 < x < 2$ or $x > 4$,
5. $f''(x) > 0$ if $1 < x < 3, \quad f''(x) < 0$ if $x < 1$ or $x > 3$.

Solution: `ww2.coastal.edu/rdahal/math160/ap3/`

[8] 10. Consider the function $f(x) = x^3 - 6x^2$. Answer the following using **calculus**.

(a) Find the intervals on which f is increasing or decreasing.

$f'(x) = 3x^2 - 12x = 0$
$\Rightarrow 3x(x-4) = 0$
$x = 0, 4$

f': $-\infty$ + + | 0 | − − | 4 | + + ∞
f: ↗ 0 ↘ 4 ↗

f is increasing on $(-\infty, 0) \cup (4, \infty)$; decreasing on $(0, 4)$.

(b) Find the x−values where f attains its local maximum and minimum values.

Local max. at $x = 0$; local min. at $x = 4$.

[8] 11. Consider the function $f(x) = x^4 - 6x^3$. Answer the following using **calculus**.

(a) Find the intervals on which f is concave up or concave down.

$f'(x) = 4x^3 - 18x^2$
$f''(x) = 12x^2 - 36x = 0$
$\Rightarrow 12x(x-3) = 0$
$x = 0, 3$

f'': $-\infty$ + + | 0 | − − | 3 | + + ∞
f: ∪ 0 ∩ 3 ∪

f is concave up on $(-\infty, 0)$ and $(3, \infty)$; concave down on $(0, 3)$.

(b) Find the x− coordinate(s) of inflection point(s) of f.

$x = 0, 3$

Circle the correct answer. You do not need to show your work. (No partial credit will be given.)

[3] 12. If $f(x) = e^{2x}$. Find the value of $f''(0)$.

(a) 1 (b) $4 \ln 2$ (c) 2 (d) $2e^{2x}$ **[(e) 4]**

$f'(x) = 2e^{2x}$
$f''(x) = 4e^{2x}$
$4 \cdot e^0 = 4$

[3] 13. The position of a bird flying along a straight line in t seconds is given by $s(t) = 3t^3 - 7t$ meters. What is the acceleration (in $meters/sec^2$) after 1 second?

(a) 2 (b) −4 (c) 9 **[(d) 18]** (e) 11

$v(t) = 9t^2 - 7$
$a(t) = 18t$
$18 \cdot 1$

Use the table for the following two questions.

$f(0)$	$f'(0)$	$g(0)$	$g'(0)$
0	-2	4	2

[3] 14. If $H(x) = g(f(x)) + 1$, find $H'(0)$.

(a) 2 **[(b) −4]** (c) 5 (d) −2 (e) 3

$H'(x) = g'(f(x)) \cdot f'(x)$
$H'(0) = g'(f(0)) \cdot f'(0)$
$= g'(0) \cdot (-2)$

[3] 15. If $J(x) = 5^{f(x)}$, find $J'(0)$.

$J'(x) = 5^{f(x)} \cdot \ln 5 \cdot f'(x)$

(a) 0 (b) $2 \ln 5$ **[(c) $-2 \ln 5$]** (d) $\frac{2}{\ln 5}$ (e) $-\frac{2}{\ln 5}$