

Name: \_\_\_\_\_

Please read each question carefully to ensure that you are actually answering it. **Show all work.**  
All numerical answers should be left in **exact** form unless otherwise specified.

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Points:	16	22	18	16	16	12	100
Score:							

16 1. Find the derivatives of the following functions. You do not need to simplify your answer.

(a)  $f(x) = (1 + \tan^{-1} x)^3$

(b)  $f(x) = \sqrt{\frac{x^2 - 5}{3x + 2}}$

(c)  $f(x) = \sin^2(3x)$

(d)  $f(x) = \ln(4x^3 + x)$

8 2. Find  $\frac{dy}{dx}$  by implicit differentiation.  $\cos(x^2y^3) = 5x^2 - 2$

8 3. Find an **equation of the tangent line** to the curve  $y = \sin(\tan x)$  at  $x = 0$ .

4. A ball is thrown upward with 80 ft/sec of velocity from a 64-foot-tall building. After  $t$  seconds, its height above the ground is given by  $s(t) = -16t^2 + 80t + 64$ .

2 (a) What is the **velocity** of the ball after 2 sec?

4 (b) What is the **maximum height** reached by the ball?

- 6 5. Determine the absolute maximum and minimum of  $f$  on the given interval.

$$f(x) = x^2 - 4, \quad [-1, 4]$$

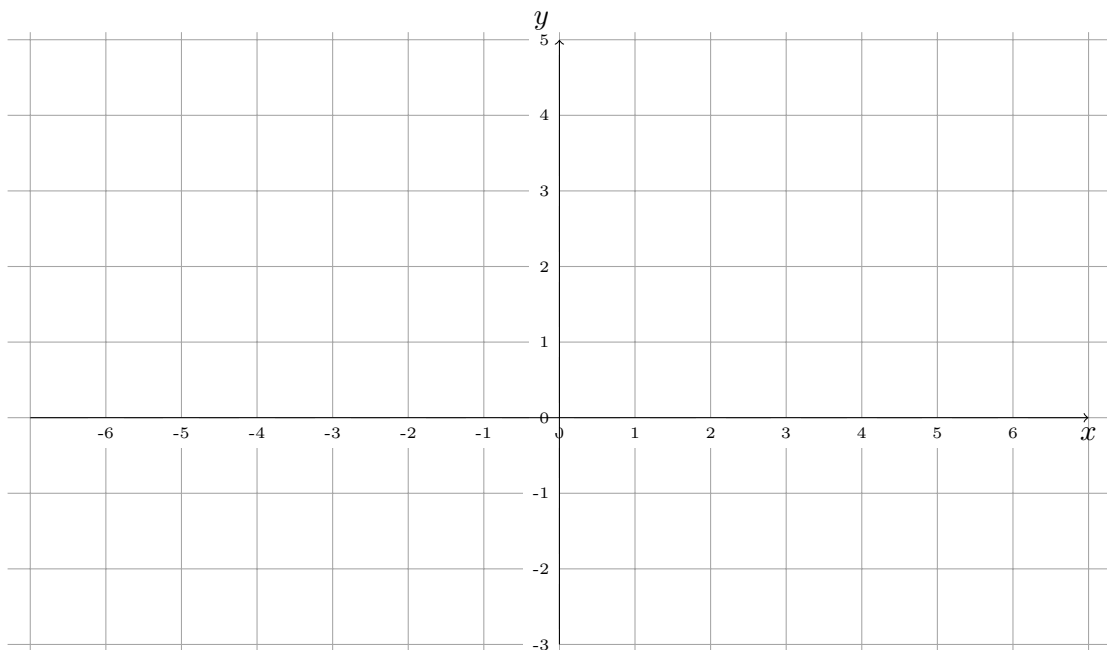
- 6 6. Find the linear approximation  $L(x)$  to  $f(x) = \sqrt{x}$  at  $x = 9$  and use the approximation to estimate  $\sqrt{9.1}$ .

- 6 7. Given the function  $f(x) = x^3$  on the interval  $[0, 2]$ , determine if the Mean Value Theorem applies. If so, find all points  $c$  which satisfy the conclusion of the theorem. If not, explain why the theorem doesn't apply.

- 9 8. A balloon is rising at a constant speed of 5 ft per sec. A boy is cycling along a straight road at a speed of 15 ft per sec. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 sec later?

- 7 9. Sketch the graph of a function  $f$  that satisfies all of the given conditions.

1.  $f(0) = 4$ ,  $f(2) = -2$ , and  $f(4) = 1$ ,
2.  $f'(0) = f'(2) = f'(4) = 0$ ,
3.  $f'(x) > 0$  if  $x < 0$  or  $2 < x < 4$ ,
4.  $f'(x) < 0$  if  $0 < x < 2$  or  $x > 4$ ,
5.  $f''(x) > 0$  if  $1 < x < 3$ ,  $f''(x) < 0$  if  $x < 1$  or  $x > 3$ .



- 8 10. Consider the function  $f(x) = x^3 - 6x^2$ . Answer the following using **calculus**. (Answer produced from graphing calculator receives no credit.)

(a) Find the intervals on which  $f$  is increasing or decreasing. Show your work to support your answer. Answer in the space provided.

$f$  is increasing on .....

$f$  is decreasing on .....

(b) Find the  $x$ -values where  $f$  attains its local maximum and minimum values.

$f$  has local maximum at  $x = \dots\dots$

$f$  has local minimum at  $x = \dots\dots$

- 8 11. Consider the function  $f(x) = x^4 - 6x^3$ . Answer the following using **calculus**. (Answer produced from graphing calculator receives no credit.)

(a) Find the intervals on which  $f$  is concave up or concave down.

$f$  is concave up on .....

$f$  is concave down on .....

(b) Find the  $x$ - coordinate(s) of inflection point(s) of  $f$ .

$f$  has inflection point(s) at  $x = \dots\dots\dots$

Circle the correct answer. You do not need to show your work. (No partial credit will be given.)

3 12. If  $f(x) = e^{2x}$ . Find the value of  $f''(0)$ .

- (a) 1      (b)  $4 \ln 2$       (c) 2      (d)  $2e^{2x}$       (e) 4

3 13. The position of a bird flying along a straight line in  $t$  seconds is given by  $s(t) = 3t^3 - 7t$  meters. What is the acceleration (in *meters/sec*<sup>2</sup>) after 1 second?

- (a) 2      (b)  $-4$       (c) 9      (d) 18      (e) 11

Use the table for the following two questions.

$f(0)$	$f'(0)$	$g(0)$	$g'(0)$
0	-2	4	2

3 14. If  $H(x) = g(f(x)) + 1$ , find  $H'(0)$ .

- (a) 2      (b)  $-4$       (c) 5      (d)  $-2$       (e) 3

3 15. If  $J(x) = 5^{f(x)}$ , find  $J'(0)$ .

- (a) 0      (b)  $2 \ln 5$       (c)  $-2 \ln 5$       (d)  $\frac{2}{\ln 5}$       (e)  $-\frac{2}{\ln 5}$