

Test 1 Integral Review

Hints for which method to use are on the next page. Answers and then full solutions follow. (Please give the integrals a try first before consulting the Hints/Answers/Solutions!

1. $\int \tan^{-1}(x) dx$

2. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$

3. $\int_1^2 \frac{1}{x^2 \sqrt{x^2 - 1}} dx$

4. $\int \frac{1}{x^4 - x^3} dx$

5. $\int_0^{\pi/2} \cos^3(x) \sqrt{\sin(x)} dx$

6. $\int \sqrt{3 - x^2} dx$

7. $\int_0^{\pi/2} x^2 \cos(x) dx$

8. $\int \sec(x) \tan^3(x) dx$

9. $\int \frac{3x^2 - 2}{x^2 - 2x - 8} dx$

10. $\int_0^1 \frac{1}{(x^2 + 1)(x + 1)} dx$

11. $\int_1^{e^\pi} \frac{\cos(\ln|x|)}{x} dx$

12. $\int \frac{1}{(x^2 + 4)^{3/2}} dx$

13. $\int \cos^4(2x) dx$

14. $\int_1^2 x^2 e^{3x} dx$

15. $\int e^{\sin 4x} \cos 4x dx$

Hint: For one of trig sub problems, you'll need that $\sin(2\theta) = 2 \sin \theta \cos \theta$.

Hints on suggested methods

1. integration by parts (after parts, should require u-sub)
2. u-sub
3. trig substitution
4. partial fractions
5. trig integral
6. trig substitution (after substitution, should have a trig integral; after integrating, here's where the hint about double angle formula on the last page comes in.)
7. integration by parts (will have to apply twice)
8. trig integral
9. long division (should follow with partial fractions)
10. partial fractions (one or two of your subsequent integrals should require u-sub)
11. u-sub
12. trig substitution
13. trig integral
14. integration by parts (will have to apply twice)
15. u-sub

Answers

1. $x \tan^{-1}(x) - \frac{1}{2} \ln |x^2 + 1| + C$
2. $-2 \cos(\sqrt{x}) + C$
3. $\sqrt{3}/2$
4. $\frac{1}{x} + \frac{1}{2x^2} + \ln \left| \frac{x-1}{x} \right| + C$
5. $\frac{8}{21}$
6. $\frac{3}{2} \sin^{-1} \left(\frac{x}{\sqrt{3}} \right) + \frac{x\sqrt{3-x^2}}{2} + C$
7. $\frac{\pi^2}{4} - 2$
8. $\frac{\sec^3(x)}{3} - \sec(x) + C$
9. $3x + \frac{23}{3} \ln |x-4| - \frac{5}{3} \ln |x+2| + C$
10. $\frac{1}{4} \ln(2) + \frac{\pi}{8}$
11. 0
12. $\frac{1}{4} \frac{x}{\sqrt{x^2+4}} + C$
13. $\frac{3}{8}x + \frac{1}{8} \sin 4x + \frac{\sin 8x}{64} + C$
14. $\frac{1}{27}e^3 [26e^3 - 5]$
15. $\frac{e^{\sin 4x}}{4} + C$

Full solutions

$$\begin{aligned} 1. \quad & \int \tan^{-1}(x) dx \\ &= \int 1 \cdot \tan^{-1}(x) dx \end{aligned}$$

Try integration by parts with $u = \tan^{-1}(x)$ and $dv = 1 dx$. This gives $du = \frac{1}{x^2 + 1} dx$ and $v = x$.

The integration by parts formula is $\int u dv = uv - \int v du$, so our integral is:

$$= x \tan^{-1}(x) - \int \frac{x}{x^2 + 1} dx$$

We need u-sub to take care of the remaining integral. We've already assigned the variable u though, so we'll substitute with a different variable, w: Try $w = x^2 + 1$.

Then, $dw = 2x dx$, or $\frac{dw}{2} = x dx$.

$$\begin{aligned} &= x \tan^{-1}(x) - \int \frac{1}{2} \frac{1}{w} dw \\ &= x \tan^{-1}(x) - \frac{1}{2} \ln |w| + C \end{aligned}$$

Rewrite in terms of x:

$$= x \tan^{-1}(x) - \frac{1}{2} \ln |x^2 + 1| + C$$

$$2. \quad \int \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$$

There's a composition here, so try u-substitution with $u = \sqrt{x}$. Then, $du = \frac{1}{2\sqrt{x}} dx$

(by power rule), or $2du = \frac{1}{\sqrt{x}} dx$

So, our integral can be rewritten as:

$$\begin{aligned} &= \int \sin(\sqrt{x}) \cdot \frac{1}{\sqrt{x}} dx \\ &= \int \sin(u) \cdot 2du \\ &= -2 \cos(u) + C \\ &= -2 \cos(\sqrt{x}) + C \end{aligned}$$

$$3. \quad \int_1^2 \frac{1}{x^2 \sqrt{x^2 - 1}} dx$$

Don't let the composition fool you—u-sub doesn't quite out here. Since we have a difference of squares inside a square root, this is suggestive that we try trig substitution

instead. This is a secant form substitution, with $a=1$. In other words, try $x = \sec \theta$. Then, $dx = \sec \theta \tan \theta d\theta$. (Don't forget your dx !)

$$= \int \frac{\sec \theta \tan \theta}{\sec^2 \theta \sqrt{\sec^2 \theta - 1}} d\theta$$

You'll note that we did not copy over the old bounds. This is because they don't work with θ ! You'll need to use your substitution $x = \sec \theta$ to solve for the new bounds by plugging old ones in for x and solving the resulting equation for θ .

Finding 1's replacement:

$$1 = \sec \theta \text{ (take reciprocals)}$$

$$1 = \cos \theta$$

$$0 = \theta$$

Finding 2's replacement:

$$2 = \sec \theta \text{ (take reciprocals)}$$

$$1/2 = \cos \theta$$

$$\pi/3 = \theta$$

So, our integral is now:

$$= \int_0^{\pi/3} \frac{\sec \theta \tan \theta}{\sec^2 \theta \sqrt{\sec^2 \theta - 1}} d\theta$$

$$= \int_0^{\pi/3} \frac{\tan \theta}{\sec \theta \sqrt{\sec^2 \theta - 1}} d\theta$$

Since this was a trig sub problem, we should be able to apply a Pythagorean trig identity to rewrite the inside of the radical:

$$= \int_0^{\pi/3} \frac{\tan \theta}{\sec \theta \sqrt{\tan^2 \theta}} d\theta$$

$$= \int_0^{\pi/3} \frac{\tan \theta}{\sec \theta \tan \theta} d\theta$$

$$= \int_0^{\pi/3} \frac{1}{\sec \theta} d\theta$$

$$= \int_0^{\pi/3} \cos \theta d\theta$$

$$= \sin \theta \Big|_0^{\pi/3}$$

Since we changed our bounds over, we can evaluate this directly and don't need to bother with rewriting in terms of x .

$$= \sin(\pi/3) - \sin(0)$$

$$= \sqrt{3}/2$$

$$4. \int \frac{1}{x^4 - x^3} dx$$

The bottom factors nicely—try partial fractions.

$$\int \frac{1}{x^3(x-1)} dx$$

We have a repeated linear term and a unique linear term. Set up an equation to solve for the partial fraction decomposition as follows:

$$\frac{1}{x^3(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}$$

Finding common denominators gives:

$$\frac{1}{x^3(x-1)} = \frac{Ax^2(x-1) + Bx(x-1) + C(x-1) + Dx^3}{x^3(x-1)}$$

At this point we may examine numerators only:

$$1 = Ax^2(x-1) + Bx(x-1) + C(x-1) + Dx^3$$

Plugging in $x=1$ is a shortcut that will give us $D = 1$; and plugging in $x=0$ is a shortcut that will give us $-C = 1$, or $C = -1$.

$$1 = Ax^2(x-1) + Bx(x-1) - x + 1 + x^3$$

We can expand the RHS and check coefficients to try to solve a system of equations for the remaining variables A and B.

$$1 = Ax^3 - Ax^2 + Bx^2 - Bx - x + 1 + x^3.$$

This suggests $0 = A + 1$, or $A = -1$ by looking at the coefficients on the cube terms. This also suggests $0 = -B - 1$, or $B = -1$ if we look at the coefficients on the x 's.

Thus, our integral can be rewritten as:

$$\begin{aligned} &= \int -\frac{1}{x} - \frac{1}{x^2} - \frac{1}{x^3} + \frac{1}{x-1} dx \\ &= \int -\frac{1}{x} - x^{-2} - x^{-3} + \frac{1}{x-1} dx \\ &= -\ln|x| + x^{-1} + \frac{x^{-2}}{2} + \ln|x-1| + C \\ &= -\ln|x| + \frac{1}{x} + \frac{1}{2x^2} + \ln|x-1| + C, \text{ or} \\ &= \frac{1}{x} + \frac{1}{2x^2} + \ln\left|\frac{x-1}{x}\right| + C \end{aligned}$$

$$5. \int_0^{\pi/2} \cos^3(x) \sqrt{\sin(x)} dx$$

Cosine appears to an odd power here, so consider a trig integral. We can reserve one of the copies of cosine for use in the du of a u -substitution, but then we'll have to rewrite the rest of the cosines in terms of sine using the Pythagorean trig identity.

$$\begin{aligned}
&= \int_0^{\pi/2} \cos^2(x) \sqrt{\sin(x)} \cos(x) dx \\
&= \int_0^{\pi/2} (1 - \sin^2(x)) \sqrt{\sin(x)} \cos(x) dx
\end{aligned}$$

Try $u = \sin(x)$. Then, $du = \cos(x)dx$. Note that the u-sub changes bounds—we plug the old ones in for x in $u = \sin(x)$ to get the new bounds on u :

$$= \int_0^1 (1 - u^2) \sqrt{u} du$$

Try expanding before integrating here.

$$\begin{aligned}
&= \int_0^1 (1 - u^2) u^{1/2} du \\
&= \int_0^1 (u^{1/2} - u^{5/2}) du \\
&= \left[\frac{u^{3/2}}{3/2} - \frac{u^{7/2}}{7/2} \right]_0^1 \\
&= \left[\frac{2u^{3/2}}{3} - \frac{2u^{7/2}}{7} \right]_0^1 \\
&= \left[\frac{2}{3} - \frac{2}{7} \right] - [0 - 0] \\
&= \frac{14}{21} - \frac{6}{21} \\
&= \frac{8}{21}
\end{aligned}$$

6. $\int \sqrt{3 - x^2} dx$

We have a difference of squares inside a square root. This means we probably need trig substitution. Here, try $x = \sqrt{3} \sin \theta$, so that $dx = \sqrt{3} \cos \theta d\theta$.

$$= \int \sqrt{3 - 3 \sin^2 \theta} \cdot \sqrt{3} \cos \theta d\theta$$

We'll need to factor the inside of the radical to apply the necessary trig identity to simplify.

$$\begin{aligned}
&= \int \sqrt{3(1 - \sin^2 \theta)} \cdot \sqrt{3} \cos \theta d\theta \\
&= \int \sqrt{3(\cos^2 \theta)} \cdot \sqrt{3} \cos \theta d\theta \\
&= \int \sqrt{3} \cos \theta \cdot \sqrt{3} \cos \theta d\theta
\end{aligned}$$

$$= \int 3 \cos^2 \theta d\theta$$

Note that this is a trig integral now, with even powers only. Rewrite using the half angle identity.

$$\begin{aligned} &= \int 3 \left(\frac{1}{2} + \frac{\cos 2\theta}{2} \right) d\theta \\ &= \int \frac{3}{2} + \frac{3 \cos 2\theta}{2} d\theta \\ &= \frac{3}{2}\theta + \frac{3 \sin 2\theta}{4} + C \end{aligned}$$

Now, we need to rewrite this in terms of x . Note that we can't quite set up a reference triangle or use trig identities to rewrite the sine term with the double angle—so here's where the hint from the first page comes in. Rewrite the sine term with the double angle identity:

$$\begin{aligned} &= \frac{3}{2}\theta + \frac{3 \cdot 2 \sin \theta \cos \theta}{4} + C \\ &= \frac{3}{2}\theta + \frac{3 \sin \theta \cos \theta}{2} + C \end{aligned}$$

We can use that $x = \sqrt{3} \sin \theta$, or $\frac{x}{\sqrt{3}} = \sin \theta$ to solve directly for θ using the arcsine function. We may pair this substitution with the Pythagorean trig identity to solve for $\cos \theta$ as follows:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{x^2}{3} + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \frac{x^2}{3}$$

$$\cos^2 \theta = \frac{3-x^2}{3}$$

$$\cos \theta = \sqrt{\frac{3-x^2}{3}}$$

So, rewriting our integral now should give us:

$$\begin{aligned} &= \frac{3}{2} \sin^{-1} \left(\frac{x}{\sqrt{3}} \right) + \frac{3 \cdot \frac{x}{\sqrt{3}} \cdot \sqrt{\frac{3-x^2}{3}}}{2} + C \\ &= \frac{3}{2} \sin^{-1} \left(\frac{x}{\sqrt{3}} \right) + \frac{x\sqrt{3-x^2}}{2} + C \end{aligned}$$

7. $\int_0^{\pi/2} x^2 \cos x dx$

We have a product of functions, so try integration by parts. Let $u = x^2$, and $dv = \cos(x)dx$. Then $du = 2xdx$ and $v = \sin(x)$.

The integration by parts formula is $\int u dv = uv - \int v du$, so our integral equals:

$$= x^2 \sin(x) \Big|_0^{\pi/2} - \int_0^{\pi/2} 2x \sin(x) dx$$

We'll need to do integration by parts one more time on the remaining integral. Let's start over fresh with a new u and dv : let $u = 2x$, and $dv = \sin(x) dx$. Then, $du = 2 dx$ and $v = -\cos(x) dx$.

Applying the integration by parts formula we get:

$$\begin{aligned} &= x^2 \sin(x) \Big|_0^{\pi/2} - \left[-2x \cos(x) \Big|_0^{\pi/2} - \int_0^{\pi/2} -2 \cos(x) dx \right] \\ &= [x^2 \sin(x) + 2x \cos(x)] \Big|_0^{\pi/2} + \int_0^{\pi/2} -2 \cos(x) dx \\ &= [x^2 \sin(x) + 2x \cos(x) - 2 \sin(x)] \Big|_0^{\pi/2} \\ &= [(\pi/2)^2 \sin(\pi/2) + 2(\pi/2) \cos(\pi/2) - 2 \sin(\pi/2)] - [0 + 0 - 2 \sin(0)] \\ &= \left[\frac{\pi^2}{4} + 0 - 2 \right] - 0 \\ &= \frac{\pi^2}{4} - 2 \end{aligned}$$

8. $\int \sec(x) \tan^3(x) dx$

This is a trig integral with an odd number of tangent terms (and nonzero number of secant terms). This means we can set aside a $\sec(x) \tan(x)$ for our du , and rewrite all of the remaining tangents in terms of secant:

$$\begin{aligned} &= \int \tan^2(x) \sec(x) \tan(x) dx \\ &= \int (\sec^2(x) - 1) \sec(x) \tan(x) dx \end{aligned}$$

Now we can try a u -sub with $u = \sec(x)$ and $du = \sec(x) \tan(x) dx$:

$$\begin{aligned} &= \int (u^2 - 1) du \\ &= \frac{u^3}{3} - u + C \end{aligned}$$

And, plugging u back in:

$$= \frac{\sec^3(x)}{3} - \sec(x) + C$$

9. $\int \frac{3x^2 - 2}{x^2 - 2x - 8} dx$

We've got a polynomial over a polynomial, so this implies we shoot for a method from the partial fractions section. Since the degree of the top polynomial is the same as the degree of the bottom, we start with long division:

$$x^2 - 2x - 8 \quad \left| \begin{array}{r} 3 \\ \hline 3x^2 \qquad - 2 \\ \hline 3x^2 - 6x - 24 \\ \hline 6x + 22 \end{array} \right.$$

This implies that we can rewrite the integral as:

$$\begin{aligned} &= \int 3 + \frac{6x + 22}{x^2 - 2x - 8} dx \\ &= \int 3 + \frac{6x + 22}{(x - 4)(x + 2)} dx \end{aligned}$$

Now, we can try to rewrite the rational function using partial fractions. We have two unique linear terms, so we set up:

$$\frac{6x + 22}{(x - 4)(x + 2)} = \frac{A}{x - 4} + \frac{B}{x + 2}$$

Find the common denominator and add on the right to get:

$$\frac{6x + 22}{(x - 4)(x + 2)} = \frac{A(x + 2) + B(x - 4)}{(x - 4)(x + 2)}$$

And now we can look at only the numerators:

$$6x + 22 = A(x + 2) + B(x - 4)$$

At this point, we can plug in $x = -2$ to get that

$$-12 + 22 = A \cdot 0 + B \cdot (-6)$$

$$10 = -6B$$

$$B = -5/3$$

We can similarly plug in $x = 4$ instead to get that

$$24 + 22 = A \cdot 6 + B \cdot 0$$

$$46 = 6A$$

$$A = 23/3.$$

We can now rewrite our integrand and integrate:

$$\begin{aligned} &= \int 3 + \frac{A}{x - 4} - \frac{B}{x + 2} dx \\ &= \int 3 + \frac{23/3}{x - 4} - \frac{5/3}{x + 2} dx \\ &= 3x + \frac{23}{3} \ln|x - 4| - \frac{5}{3} \ln|x + 2| + C \end{aligned}$$

$$10. \int_0^1 \frac{1}{(x^2 + 1)(x + 1)} dx$$

We've got a rational function with no clear u-sub. The bottom is already factored in order for us to try partial fractions. So, set up the function for rewriting with a unique linear term and an irreducible quadratic term. We'd like it to look like the following:

$$\frac{1}{(x^2 + 1)(x + 1)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 1}$$

Find a common denominator and add:

$$\frac{1}{(x^2 + 1)(x + 1)} = \frac{(Ax + B)(x + 1) + C(x^2 + 1)}{(x^2 + 1)(x + 1)}$$

Now we can look at the numerators only:

$$1 = (Ax + B)(x + 1) + C(x^2 + 1)$$

We can try plugging in $x = -1$ to solve for one of the variables:

$$1 = (-A + B) \cdot 0 + C(1 + 1)$$

$$1 = 2C, \text{ so } C = \frac{1}{2}.$$

This gives us the equation:

$$1 = (Ax + B)(x + 1) + \frac{1}{2}(x^2 + 1)$$

We can try plugging in $x = 0$ now to get rid of the A term so that we can solve for B:

$$1 = (A \cdot 0 + B)(1) + \frac{1}{2}(1)$$

$$1 = B + \frac{1}{2}, \text{ so } B \text{ also is equal to } \frac{1}{2}.$$

Now, we can try multiplying out and checking coefficients in order to solve for B:

$$1 = (Ax + \frac{1}{2})(x + 1) + \frac{1}{2}(x^2 + 1)$$

$$1 = Ax^2 + Ax + \frac{1}{2}x + \frac{1}{2} + \frac{1}{2}x^2 + \frac{1}{2}$$

Notice that the coefficients on both the x^2 and x terms implies that $A + \frac{1}{2} = 0$. Thus, $A = -\frac{1}{2}$.

Now that we have all of the coefficients figured out, we can rewrite the integral:

$$\begin{aligned} &= \int_0^1 \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 1} dx \\ &= \int_0^1 \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2 + 1} + \frac{\frac{1}{2}}{x + 1} dx \\ &= \frac{1}{2} \int_0^1 \frac{-x + 1}{x^2 + 1} + \frac{1}{x + 1} dx \end{aligned}$$

We can split the integral into pieces to make each easier to handle. This will give us

$$= \frac{1}{2} \left[\int_0^1 \frac{-x}{x^2 + 1} dx + \int_0^1 \frac{1}{x^2 + 1} dx + \int_0^1 \frac{1}{x + 1} dx \right]$$

The first integral will require a u-substitution, using $u = x^2 + 1$ and $du = 2xdx$, or $du/2 = xdx$.

$$= \frac{1}{2} \left[\int_{u(0)}^{u(1)} \frac{-1}{u} \cdot \frac{1}{2} du + \int_0^1 \frac{1}{x^2 + 1} dx + \int_0^1 \frac{1}{x + 1} dx \right]$$

These are now reasonably straightforward integrals to compute:

$$= \frac{1}{2} \left(\left[-\frac{1}{2} \ln |u| \right]_{u(0)}^{u(1)} + [\tan^{-1}(x) + \ln |x + 1|]_0^1 \right)$$

We can rewrite the u term in terms of x so that we can evaluate the bounds all at the same time.

$$\begin{aligned} &= \frac{1}{2} \left[-\frac{1}{2} \ln |x^2 + 1| + \tan^{-1}(x) + \ln |x + 1| \right]_0^1 \\ &= \frac{1}{2} \left(\left[-\frac{1}{2} \ln(2) + \tan^{-1}(1) + \ln(2) \right] - \left[-\frac{1}{2} \ln(1) + \tan^{-1}(0) + \ln(1) \right] \right) \\ &= \frac{1}{2} \left(\left[-\frac{1}{2} \ln(2) + \frac{\pi}{4} + \ln(2) \right] - [0 + 0 + 0] \right) \\ &= \frac{1}{2} \left(\frac{1}{2} \ln(2) + \frac{\pi}{4} \right) \\ &= \frac{1}{4} \ln(2) + \frac{\pi}{8} \end{aligned}$$

$$\begin{aligned} 11. \int_1^{e^\pi} \frac{\cos(\ln |x|)}{x} dx \\ &= \int_1^{e^\pi} \cos(\ln |x|) \cdot \frac{1}{x} dx \end{aligned}$$

We have a composition, so let's try u-sub. Let $u = \ln |x|$, so that $du = \frac{1}{x} dx$. Then, we may rewrite the integral in terms of u—which includes changing our old bounds over by plugging them into the u we selected.

$$\begin{aligned} &= \int_{\ln 1}^{\ln e^\pi} \cos u \, du \\ &= \int_0^\pi \cos u \, du \\ &= \sin(u) \Big|_0^\pi \\ &= \sin(\pi) - \sin(0) \\ &= 0 \end{aligned}$$

$$12. \int \frac{1}{(x^2 + 4)^{3/2}} dx$$

The form of this integral suggests that we use a trig substitution. Try $x = 2 \tan \theta$, so that $dx = 2 \sec^2 \theta d\theta$. Now when we plug things in, we get:

$$= \int \frac{2 \sec^2 \theta}{(4 \tan^2 \theta + 4)^{3/2}} d\theta$$

Now we can factor a 4 out in the denominator so we can simplify with a trig identity:

$$= \int \frac{2 \sec^2 \theta}{(4)^{3/2} (\tan^2 \theta + 1)^{3/2}} d\theta$$

$$= \int \frac{2 \sec^2 \theta}{(4)^{3/2} (\sec^2 \theta)^{3/2}} d\theta$$

$$= \int \frac{2 \sec^2 \theta}{(4)^{3/2} \sec^3 \theta} d\theta$$

$$= \int \frac{2 \sec^2 \theta}{(\sqrt{4})^3 \sec^3 \theta} d\theta$$

$$= \int \frac{2 \sec^2 \theta}{(2)^3 \sec^3 \theta} d\theta$$

$$= \int \frac{1}{4 \sec \theta} d\theta$$

$$= \int \frac{1}{4} \cos \theta d\theta$$

$$= \frac{1}{4} \sin \theta + C$$

Now we need to rewrite $\sin \theta$ in terms of x . Use the fact that $x = 2 \tan \theta$ was our trig sub, which means $\frac{x}{2} = \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$. If we set up a reference triangle, we'll find out that the hypotenuse is of length $\sqrt{x^2 + 4}$. Then, we get $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{\sqrt{x^2 + 4}}$.

Our integral then comes out to:

$$= \frac{1}{4} \frac{x}{\sqrt{x^2 + 4}} + C$$

13. $\int \cos^4(2x) dx$

This is an integral of a trig function with only even powers. This means we'll want to rewrite it using the half angle formula $\cos^2 \theta = \frac{1}{2} + \frac{\cos 2\theta}{2}$. Rewrite the cosine term as cosine squared raised to some power to make it easier to plug in.

$$= \int [\cos^2(2x)]^2 dx$$

$$= \int \left[\frac{1}{2} + \frac{\cos 4x}{2} \right]^2 dx$$

$$= \int \frac{1}{4} + \frac{1}{2} \cos 4x + \frac{\cos^2 4x}{4} dx$$

Note that we have another even trig integral, so we'll have to apply the identity again.

$$\begin{aligned}
 &= \int \frac{1}{4} + \frac{1}{2} \cos 4x + \frac{1}{4} \left[\frac{1}{2} + \frac{\cos 8x}{2} \right] dx \\
 &= \int \frac{1}{4} + \frac{1}{2} \cos 4x + \frac{1}{8} + \frac{\cos 8x}{8} dx \\
 &= \int \frac{3}{8} + \frac{1}{2} \cos 4x + \frac{\cos 8x}{8} dx \\
 &= \frac{3}{8}x + \frac{1}{8} \sin 4x + \frac{\sin 8x}{64} + C
 \end{aligned}$$

14. $\int_1^2 x^2 e^{3x} dx$

We have a product of two functions. We'll need integration by parts. Try $u = x^2$ and $dv = e^{3x} dx$, so that $du = 2x dx$ and $v = \frac{e^{3x}}{3}$.

The integration by parts formula is $\int u dv = uv - \int v du$, so our integral can be rewritten:

$$= \frac{x^2 e^{3x}}{3} \Big|_1^2 - \int_1^2 \frac{2}{3} x e^{3x} dx$$

We again need integration by parts on the remaining integral, so let's start over with a new u and new du . Let $u = \frac{2}{3}x$ and $dv = e^{3x} dx$ so that $du = \frac{2}{3} dx$ and $v = \frac{e^{3x}}{3}$. Applying the formula one more time, we get:

$$\begin{aligned}
 &= \frac{x^2 e^{3x}}{3} \Big|_1^2 - \left(\frac{2}{9} x e^{3x} \Big|_1^2 - \int_1^2 \frac{2}{9} e^{3x} dx \right) dx \\
 &= \left[\frac{x^2 e^{3x}}{3} - \frac{2}{9} x e^{3x} \right]_1^2 + \int_1^2 \frac{2}{9} e^{3x} dx \\
 &= \left[\frac{x^2 e^{3x}}{3} - \frac{2x e^{3x}}{9} + \frac{2e^{3x}}{27} \right]_1^2 \\
 &= \left[\frac{4e^6}{3} - \frac{4e^6}{9} + \frac{2e^6}{27} \right] - \left[\frac{e^3}{3} - \frac{2e^3}{9} + \frac{2e^3}{27} \right] \\
 &= \left[\frac{36e^6}{27} - \frac{12e^6}{27} + \frac{2e^6}{27} \right] - \left[\frac{9e^3}{27} - \frac{6e^3}{27} + \frac{2e^3}{27} \right] \\
 &= \left[\frac{26e^6}{27} \right] - \left[\frac{5e^3}{27} \right] \\
 &= \frac{1}{27} e^3 [26e^3 - 5]
 \end{aligned}$$

15. $\int e^{\sin 4x} \cos 4x dx$

Notice the composition of functions, since there's a sine function inside of the exponential. We'll try a u-sub with $u = \sin 4x$ and $du = 4 \cos 4x dx$, or rather $du/4 = \cos 4x dx$. With this, the integral looks like:

$$\begin{aligned} &= \int \frac{e^u}{4} du \\ &= \frac{e^u}{4} + C \end{aligned}$$

And when we sub u back in,

$$= \frac{e^{\sin 4x}}{4} + C$$