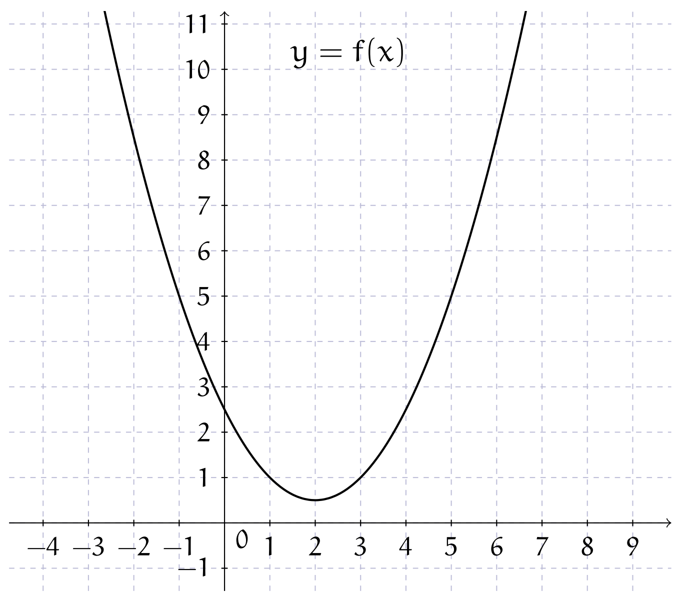
Riemann Sums

1. The graph of a function $y = f(x)$ is shown below.

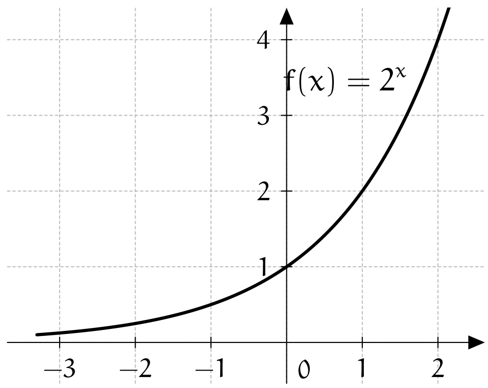
   (a) Divide the closed interval $[-1, 5]$ into 3 equal subintervals and draw the corresponding rectangles using the **left** endpoints of each subinterval.



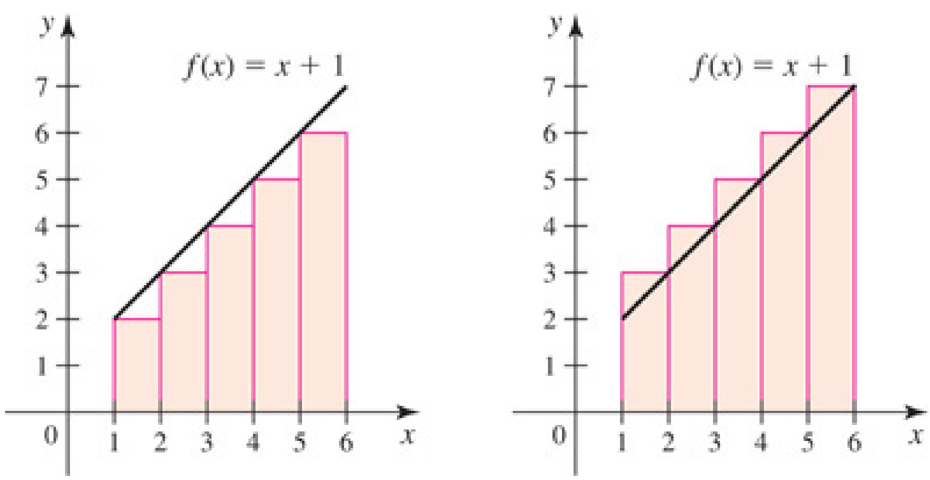
   (b) Evaluate the Riemann sum for $y = f(x)$ on the interval $[-1, 5]$ for $n = 3$, taking the sample points to be **right** endpoints.

2. The graph of $f(x) = 2^x$ is given below.

   (a) Divide the interval $[-2, 2]$ into four subintervals of equal length. Sketch the corresponding rectangles by using **right** endpoints. Then evaluate the Riemann sum.

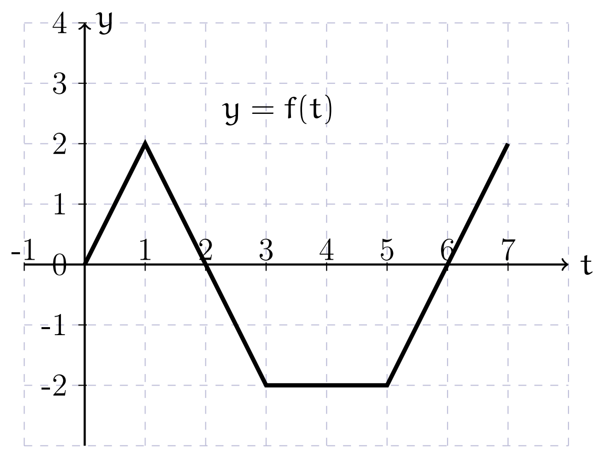


3. Use the figures to calculate the left and right Riemann sums for the function f on the interval $[1, 6]$ with $n = 5$.



Definite Integral in Terms of Areas

4. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



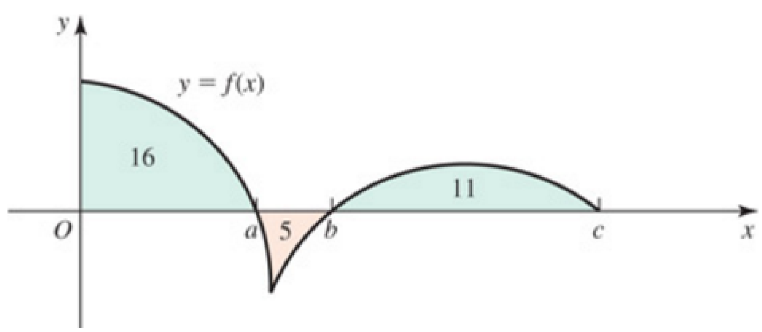
(a) $\int_0^2 f(t)\,dt$

(b) $\int_2^5 f(t)\,dt$

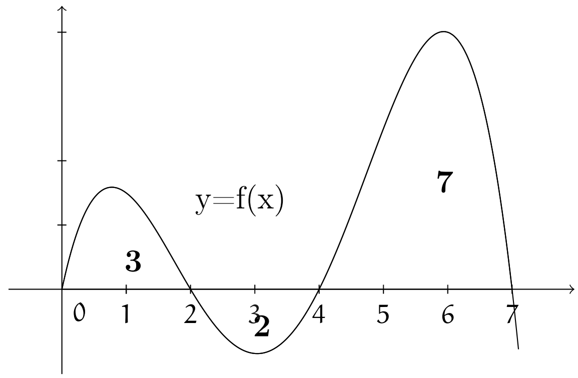
(c) $\int_5^7 f(t)\,dt$

(d) $\int_0^7 f(t)\,dt$

5. The figure shows the areas of regions bounded by the graph of f and the x−axis. Evaluate $\int_0^c f(x)dx$.

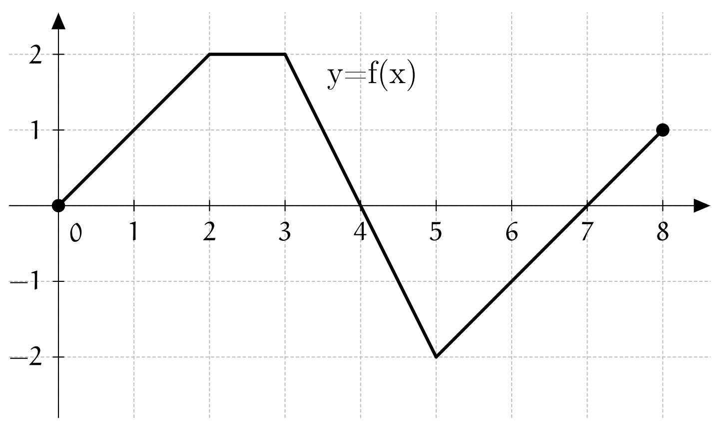


6. In the following figure the areas of the shaded regions are 3, 2, and 7 unit squares respectively.



What is the value of $\int_0^7 |f(x)|\,dx - \int_0^4 f(x)\,dx$?

7. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



(a) $\int_0^4 f(x)\,dx$

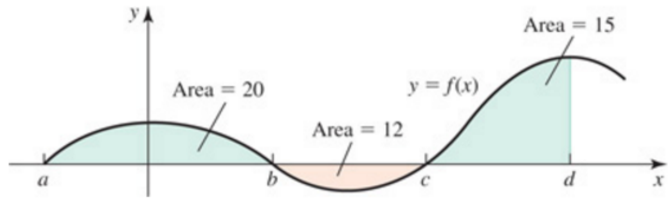
(b) $\int_3^5 f(x)\,dx$

(c) $\int_4^7 f(x)\,dx$

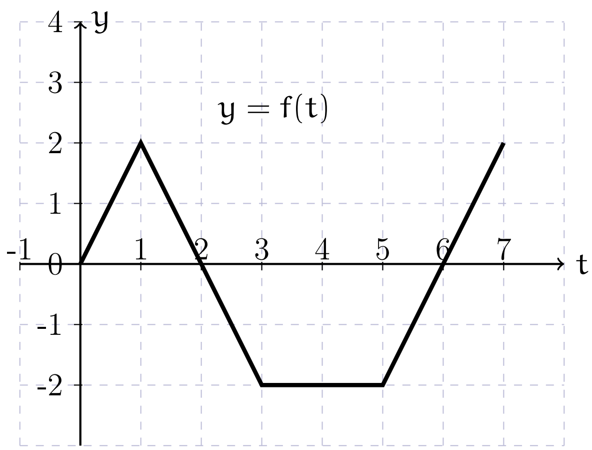
(d) $\int_0^8 f(x)\,dx$

8. Consider the following graph of $y = f(x)$.

Evaluate: $\int_a^d f(x)\,dx - \int_c^b f(x)\,dx$



9. Let $g(x) = \int_0^x f(t)dt$ where f is the function whose graph is given as below and x is in $[0, 7]$.

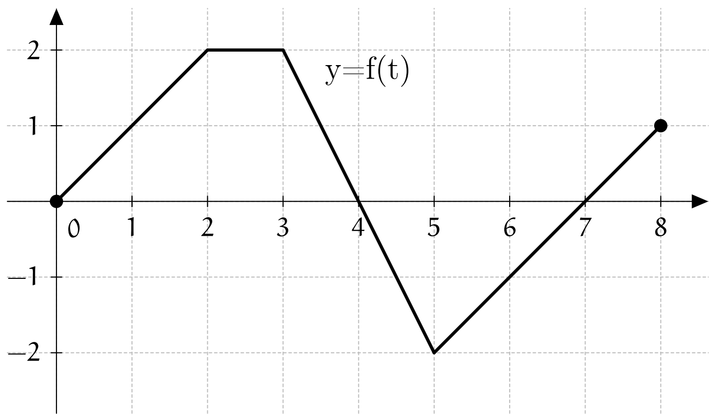


(a) On what interval is g increasing and decreasing? Briefly explain.

(b) Where does g have a maximum value?

10. Define $g(x) = \int_0^x f(t)\,dt$ where f is the function whose graph is given below and x is in $[0, 7]$.

On what interval(s) is g decreasing? Briefly explain.



11. Evaluate the following integrals by interpreting them in terms of areas.

(a) $\int_{-1}^{2} |x|\,dx$ (Hint: use the graph of $f(x) = |x|$)

(b) Evaluate $\int_0^1 \sqrt{1-x^2}\,dx$ (Hint: use the graph of $f(x) = \sqrt{1-x^2}$)

12. If $\int_1^5 f(x)dx = 10$, and $\int_3^5 f(x)dx = 5.7$, find the value of $\int_3^1 f(x)dx$.

13. If $\int_a^b f(x)dx = 2$, and $\int_b^c f(x)dx = 1$, find the value of $\int_a^c f(x)dx - \int_b^a f(x)dx$

Indefinite Integrals

14. Evaluate the following integrals.

    (a) $\int (x^3 - 3x^2)\ dx$

    (b) $\int (2e^x + 3\sin(x))\ dx$

    (c) $\int \frac{1}{1+x^2}\ dx$

    (d) $\int (1 - e^x)\ dx$

    (e) $\int \frac{1-\sqrt{x}+x}{x}\ dx$

    (f) $\int (x^3 - \frac{3}{x^3})\ dx$

    (g) $\int \frac{1+\cos^2(x)}{\cos^2(x)}\ dx$

    (h) $\int \frac{1}{\sqrt{1-x^2}}\ dx$

    (i) $\int (1-\sqrt{t})\ dt$

    (j) $\int \sec(\theta)\,(\sec(\theta) - \tan(\theta))\ d\theta$

    (k) $\int (z^2+1)(2z-5)\ dz$

Initial Value Problems

15. Find $f(x)$ if $f'(x) = 2x$ and $f(0) = 2$

16. Find $f(x)$ if $f'(x) = \cos(x)$ and $f\left(\frac{\pi}{2}\right) = -1$.

17. Find $f(x)$ if $f''(x) = -2 + 12x - 12x^2,\ \ f(0) = 4,\ \ f'(0) = 12$

18. A particle is moving with the given data. Find the position of the particle.

    (a) $v(t) = \sin t - \cos t,\ \ s(0) = 0$

    (b) $a(t) = 2t + 1\ \ s(0) = 3,\ \ v(0) = -2$

Fundamental Theorem of Calculus(FTC)

(FTC Part I)

19. Find the derivative, $g'(x)$, of each of the following functions.

(a) $g(x) = \displaystyle\int_x^1 \frac{\sin(2t)}{\sqrt{5t}}\, dt$,

(b) $g(x) = \displaystyle\int_2^{3x} \sqrt{1+t+t^2}\, dt$,

(c) $g(x) = \displaystyle\int_x^2 \sin(t^2+t)\, dt$,

(d) $g(x) = \displaystyle\int_2^{x^2} t^3 + t\, dt$

(FTC Part II)

Definite Integrals

20. Evaluate the following definite integrals.

(a) $\displaystyle\int_0^4 (x^3 - 2x)\, dx$

(b) $\displaystyle\int_0^{\pi/8} (1 + 2\sec^2(x))\, dx$

(c) $\displaystyle\int_{\pi/6}^{\pi/2} \csc(t)\cot(t)\, dt$

(d) $\displaystyle\int_\pi^0 \sin\theta\, d\theta$

(e) $\displaystyle\int_1^9 \frac{3x-2}{\sqrt{x}}\, dx$

(f) $\displaystyle\int_0^{\pi/4} (2 + \sec\theta\tan\theta)\, d\theta$

(g) $\displaystyle\int_1^4 2x\left(1-\sqrt{x}\right)\, dx$

(h) $\displaystyle\int_0^{\ln 2} e^x\, dx$