Math 160 **Test 3 Review**

Linerization

1. Find the linearization, $L(x)$ for $f(x) = \sqrt{x+1}$ at $a = 3$.
2. Find the linearization, $L(x)$ for $g(x) = \tan(x)$ at $a = \frac{\pi}{4}$.
3. Find the linear approximation $L(x)$ of the function $f(x) = \sqrt[3]{x}$ at $a = 64$. Use it to approximate te number $\sqrt[3]{65}$.
4. Find the linear approximation $L(x)$ to $f(x) = \sqrt{x}$ at $x = 9$ and use it to estimate $\sqrt{9.1}$.
5. Find the linearization $L(x)$ of the function $f(x) = \sin x$ at the number $a = 0$, and use it to approximate the value of $\sin\left(\frac{\pi}{72}\right)$.
6. Use a linear approximation to estimate the given number. **(Answer produced from calculator without using the linearization L(x) does not get any credit)**
   (a) $e^{0.05}$
   (b) $\sqrt[3]{8.02}$
   (c) $\ln(1.1)$
   (d) $\sin(0.1)$

Absolute Extremum

7. Determine the absolute extreme values of f on the given interval.
$$f(x) = \frac{1}{3}x^3 - 9x, \qquad [-1, 4]$$
8. Find the absolute maximum and absolute minimum values of the function
$$f(x) = 2x^3 - 24x + 101 \quad \text{on the interval } [0, 3].$$
9. Find the absolute maximum and absolute minimum values of the function f on the given interval.
$$f(x) = 2x + \frac{2}{x}, \qquad [0.5, 4]$$
10. Find the absolute maximum and minimum values of $g(x) = \sqrt{4 - x^2}, \quad -2 \le x \le 1$

Mean Value Theorem

11. Find the number(s) c that satisfies the conclusion of the Mean Value Theorem on the given interval. **Show your work.**

$$f(x) = \sqrt{x}, \quad ; [0, 16].$$

12. Find the number(s) c that satisfies the conclusion of the Mean Value Theorem on the given interval. **Show your work.**

$$f(x) = x^2 - x + 1, \quad [0, 3].$$

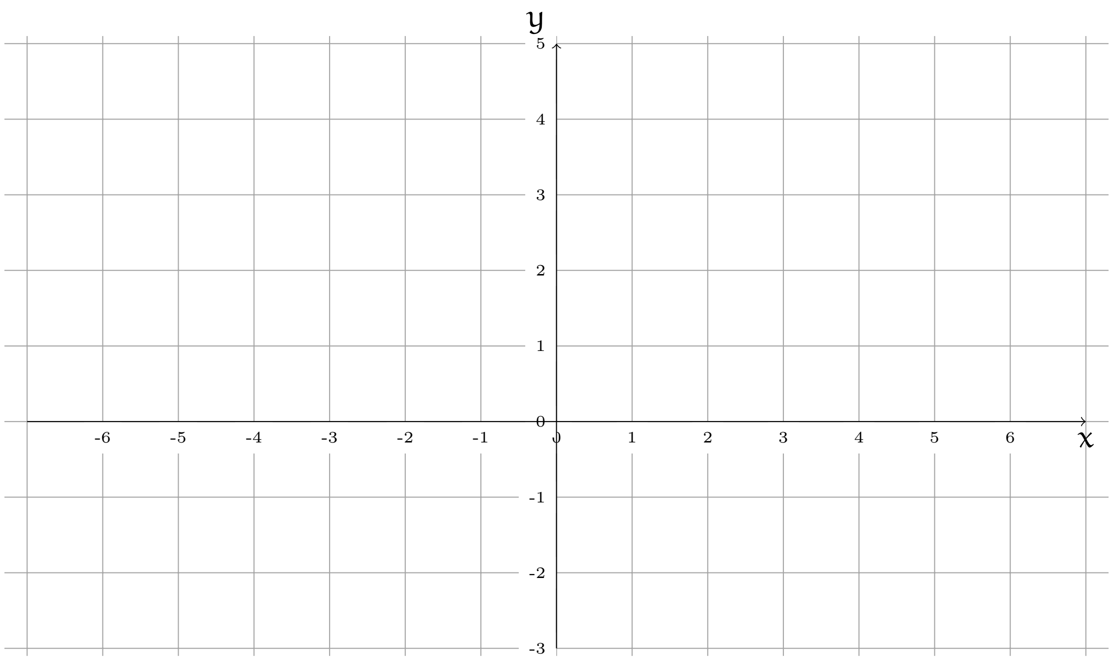
13. Let $f(x) = \frac{1}{x^2}$. Show that there is no number c in the open interval $(-1, 2)$ such that $f(2) - f(-1) = f'(c)(2 - (-1))$. Why does this not contradict the Mean Value Theorem?

14. Find the value or values of c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x + \frac{1}{X}$ on the interval $\left[\frac{1}{2}, 2\right]$

Curve sketching, Inc/Dec Local Max/Min, Concavity

15. Sketch the graph of a function f that satisfies all of the given conditions.

    1. $f(0) = 4, \quad f(2) = -2,$ and $f(4) = 1$,
    2. $f'(0) = f'(2) = f'(4) = 0$,
    3. $f'(x) > 0$ if $x < 0$ or $2 < x < 4$,
    4. $f'(x) < 0$ if $0 < x < 2$ or $x > 4$,
    5. $f''(x) > 0$ if $1 < x < 3, \quad f''(x) < 0$ if $x < 1$ or $x > 3$.



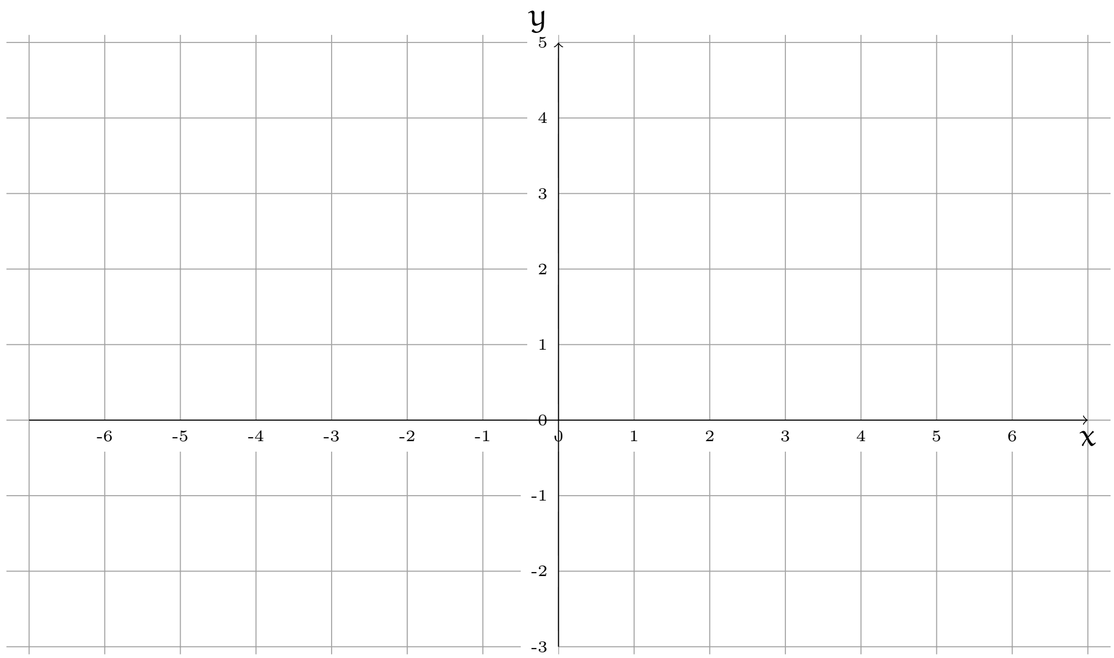
16. **Sketch a graph** of the function f that satisfies the following conditions. **Label** all intercepts, asymptotes, local maxima/minima and inflection points, if any.

    (a) The function f has only one local maximum value, $f(0) = 0$,

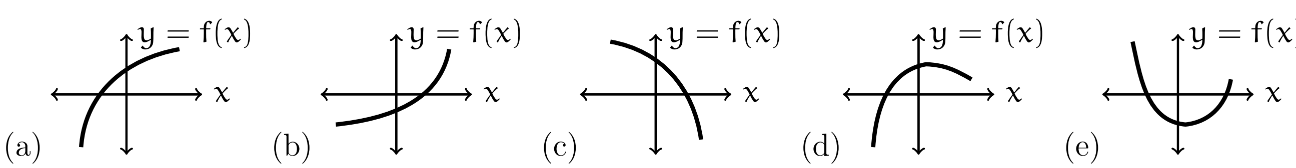
    (b) $\lim_{x\to-3^-} f(x) = \infty, \quad \lim_{x\to-3^+} f(x) = -\infty, \quad \lim_{x\to3^-} f(x) = -\infty, \quad \lim_{x\to3^+} f(x) = \infty,$ $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 1$,

    (c) $f'(x) < 0$ on the intervals $(0, 3)$ and $(3, \infty)$, $f'(x) > 0$ on the intervals $(-\infty, -3)$ and $(-3, 0)$.

    (d) $f''(x) > 0$ on the intervals $(-\infty, -3)$ and $(3, \infty)$ and $f''(x) < 0$ on the interval $(-3, 3)$.



17. If $y = f(x)$ is a function such that $f' > 0$ for all x and $f'' > 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



18. Consider the function $f(x) = 2x^3 - 3x^2$. Answer the following using **calculus**. (Answer produced from graphing calculator receives no credit.)

   (a) Find the interval(s) of increase and decrease.

   f *is increasing on* .... f *is decreasing on* ....

   (b) Use part (a) and find the $x-$values where f attains its local maximum and minimum.

   f *has local maximum at* $x =$ ...... f *has local minimum at* $x =$ ......

   (c) Find the interval(s) where the graph is concave upward and concave downward.

   f *is concave up on* .... f *is concave down on* ....

   (d) Find the $x-$ coordinate(s) of inflection point(s) of f.

   f *has inflection point(s) at* $x =$ ....

19. Consider the following function.

$$f(x) = x^3 - 3x^2 - 9x + 4.$$

   (a) Find the interval(s) of increase and decrease.

   f *is increasing on* .... f *is decreasing on* ....

   (b) Use part (a) and find the $x-$values where f attains its local maximum and minimum.

   f *has local maximum at* $x =$ ...... f *has local minimum at* $x =$ ......

   (c) Find the interval(s) where the graph is concave upward and concave downward.

   f *is concave up on* .... f *is concave down on* ....

   (d) Find the $x-$ coordinate(s) of inflection point(s) of f.

   f *has inflection point(s) at* $x =$ ....

20. Consider the function $f(x) = x^3 + 3x^2 - 9x + 10$. Answer the following using **calculus**. (Answer produced from graphing calculator receives no credit.)

(a) Find the intervals on which f is increasing or decreasing. Answer in the space provided.

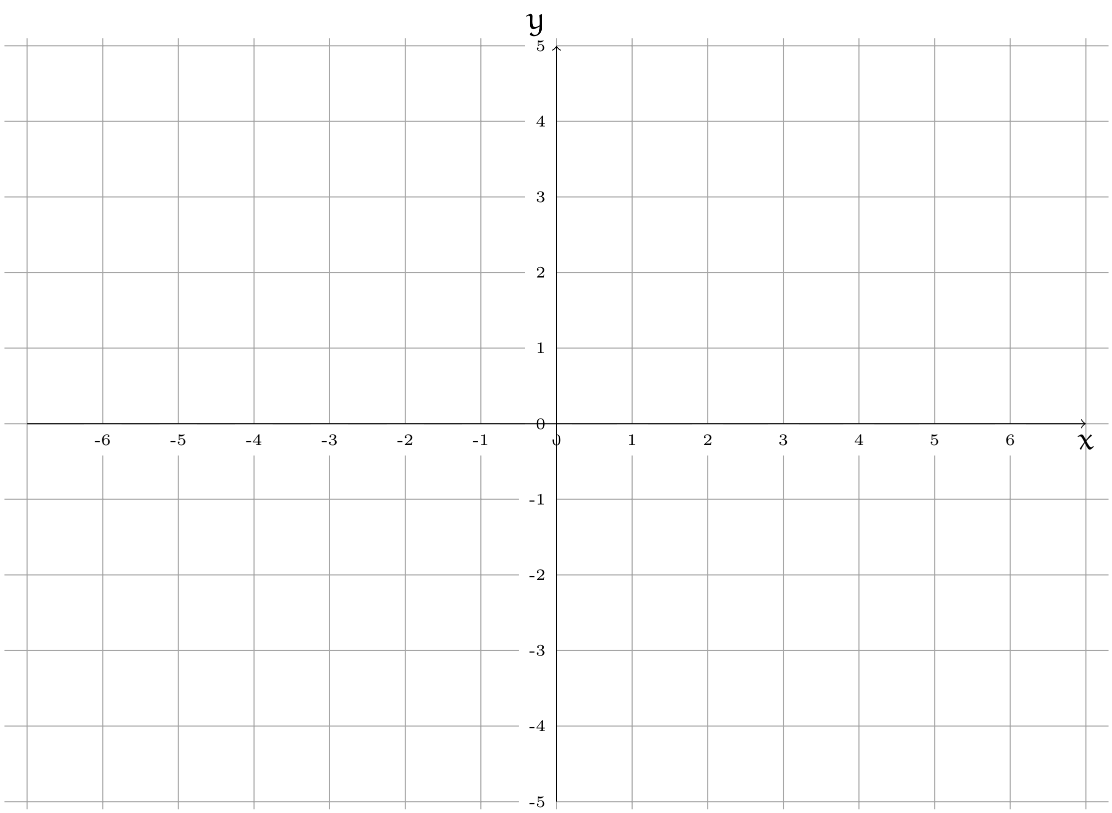
f *is increasing on* .... f *is decreasing on* ....

(b) Find the $x-$values where f attains its local maximum and minimum values.

f *has local maximum at* $x =$ ...... f *has local minimum at* $x =$ ......

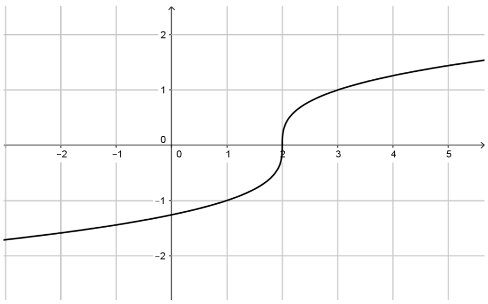
21. Sketch the graph of a function f that satisfies all of the given conditions.

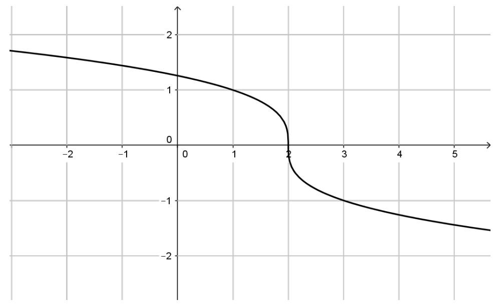
1. $f(-1) = 4, \quad f(1) = -2,$ and $f(3) = 1,$
2. $f'(-1) = f'(1) = f'(3) = 0,$
3. $f'(x) > 0$ if $x < -1$ or $1 < x < 3,$
4. $f'(x) < 0$ if $-1 < x < 1$ or $x > 3,$
5. $f''(x) > 0$ if $0 < x < 2,$
6. $f''(x) < 0$ if $x < 0$ or $x > 2.$

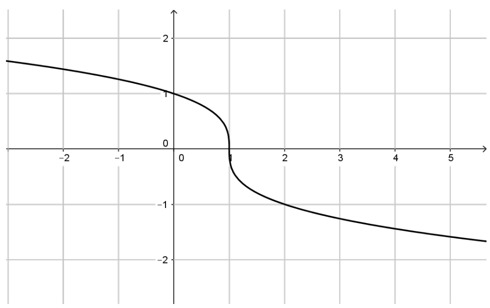


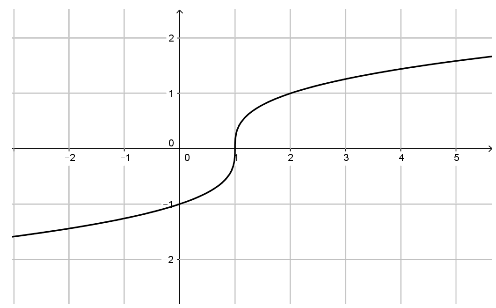
22. (3 points) **Choose a graph** of the function f that is continuous on $(-\infty, \infty)$ and has the following properties:

$$f'(x) < 0 \text{ and } f''(x) < 0 \text{ on } (-\infty, 2); \quad f'(x) < 0 \text{ and } f''(x) > 0 \text{ on } (2, \infty)$$









.

Optimization

23. A farmer has 1100 feet of fencing and he wants to build a rectangular enclosure with two pens with equal areas. Find the dimensions (length and width) that will maximize the total area of the enclosure. You must find an objective function and its derivative to answer this question.

24. A farmer has 1500 dollars available to build a fence along a straight river to create two identical rectangular pastures. The materials for the side parallel to the river cost 6 dollars per foot and the materials for the three sides perpendicular to the river cost 5 per foot. Find the dimensions for which the total area of the pastures will be as large as possible, assuming that no fence is needed along the river.

25. A box with a square base and an open top is to be constructed using 4800 sq. in. of cardboard. Find the dimensions of the box that will maximize its volume. What is the maximum volume?

26. A box with an open top is to be constructed from a square piece of cardboard, 6 feet wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

27. How close does the curve $y = \sqrt{x}$ come to the point $(3/2, 0)$? (Hint: If you minimize the square of the distance, you can avoid square roots.)

28. Scientist Sam wants to know how close a comet moving in a parabolic trajectory will get to the sun. We will assume that the sun is located at the origin, the path of the comet follows the parabola $y = x^2 - 1$ and that the units on the axes are in millions of miles. The comet is closest to the sun at two points. Find the x-coordinates of these points and the minimum distance.

L'Hopital's Rule

Evaluate the following limits. **Show all your work**

.

29. $\lim_{x \to 0} \dfrac{2\sin(4x)}{\sin(6x)}$

30. $\lim_{x \to 2} \dfrac{x^4 - 16}{x^3 - 8}$

31. $\lim_{x \to 0} \dfrac{1 - e^{2x}}{x + \sin x}$

32. $\lim_{x \to \infty} 5x^2\, e^{-2x}$

33. $\lim_{x \to \infty} x^{-2} \ln(x + 1)$

34. $\lim_{x \to -1} \dfrac{2x^2 - x - 3}{x^2 - x - 2}$

35. $\lim_{x \to 0} \dfrac{3x^2}{1 - \cos x}$

36. $\lim_{x \to \infty} x \tan(3/x)$

37. $\lim_{x \to 0} \dfrac{1 - \cos(3x)}{8x^2}$

38. $\lim_{x \to \infty} \dfrac{e^{3x}}{3e^{3x} + 5}$

39. $\lim_{x \to \infty} \dfrac{(\ln x)^2}{x}$