Calculus I Exam 2 Review

Exam Date: Friday, March 06

Position, Velocity, & Acceleration

1. The position function (in feet) of an object moving horizontally after t seconds is given by
$s(t) = 24t - 4t^2$; $0 \leq t \leq 8$, with $s > 0$ corresponding to the position right of the origin. Answer the following questions with **correct units.**
   (a) Find the velocity of the object after 1 second?
   (b) When is the object at rest, moving to the right, and moving to the left?
   (c) Determine the acceleration of the object when its velocity is zero.
   (d) Determine the time interval(s) when the object is speeding up or slowing down.

2. If a ball is thrown vertically upward with a certain velocity, its height (in feet) after t seconds is
$$s(t) = 9t - 2t^2.$$
Answer the following questions with **correct units**.
   (a) What is the **velocity** of the ball after 2 sec?
   (b) What is the **maximum height** reached by the ball?
   (c) What is the **velocity** of the ball when it is $9\,\text{ft}$ above the ground on its way down?

3. A slingshot launches a stone vertically from the top of a wall. Its height (in feet) after t seconds is given by
$$s(t) = 100 + 112t - 16t^2.$$
   (a) Find the velocity of the stone after 2 seconds.
   (b) Find the maximum height of the stone.
   (c) Find the velocity of the stone after it has risen 260 feet.
   (d) With what velocity does it hit the ground?

4. The position (in ft) of a particle moving on a straight line is given by $s(t) = t^3 - 5t^2 + t - 6$ where t is measured in seconds. What is the acceleration (in ft/sec^2) after 2 seconds?

Derivative Rules

Find the derivatives of the following functions.

5. $f(x) = 5x^2 - \frac{1}{x^2} + \sqrt[3]{x}$

6. $f(x) = 3^x + \log_3 x$

7. $f(\theta) = \sin^3(3\theta^2 - 7\theta)$

8. $f(x) = \tan\left(\frac{2x - 5}{3x + 1}\right)$

9. $f(x) = \ln(x \cot(x))$

10. $f(x) = \sin^2(3x)$

11. $f(x) = e^{3x} + \tan^{-1}(x)$

12. $f(x) = (x \sin(x))^2$

13. $f(x) = \ln\left(\frac{x^2 - 5x}{7x + 5}\right)$

14. $f(x) = 2^x + \log_5(3x)$

15. $f(x) = \ln(x \cos(x))$

16. $f(x) = \tan^3(5x)$

17. $f(x) = \left(\frac{x^3 - 7x}{5x^2 - 3x + 1}\right)^5$

18. $y = \tan^{-1}(2x)$

19. For what value of x does the graph of $f(x) = e^{2x} - 2x$ have a horizontal tangent line?

20. Find the x-value(s) of the point(s) on the curve $y = x^4 - 2x^3 + x^2 + 100$ where the tangent line is horizontal.

Implicit Differentiation

21. Find $\frac{dy}{dx}$ by implicit differentiation. $e^{x+y} = 1 + x^2y^2$

22. Find $\frac{dy}{dx}$ by implicit differentiation. $\cos(x+y) = x^2 + y^2 + x$

23. Find $\frac{dy}{dx}$ by implicit differentiation. $x^2 - 3y^2 = 2\,x\,y.$

24. Find $\frac{dy}{dx}$ by implicit differentiation. $\cos(xy) = y^2 - 5x.$

25. Find the derivative of $y = x^x$.

26. Find the derivative of $y = x^{\sin x}$.

27. Find the derivative of $y = (\sin(x))^x$

28. Find an **equation of the tangent line** to the curve $x^2 - 3y^2 = 2\,x\,y$ at the point $(1, -1)$.Express your answer in the slope -intercept form $y = mx + b$.

29. Find an **equation of the tangent line** to the curve $\cos(xy) = y^2 - 5x$ at the point $(0, 1)$. Express your answer in the slope -intercept form $y = mx + b$.

30. Evaluating Derivatives

31. If $f(x) = (2x^4 - 3x + 2)\,e^x$, find $f'(0)$.

32. $f(x) = \dfrac{5 - x^3}{4x^5 - x}$, find $f'(-1)$.

33. If $F(x) = f(xf(x))$, $f(1) = 1$, and $f'(1) = 2$, find $F'(1)$.

34. Find $G'(2)$ where $G(x) = \left(\dfrac{x^2+1}{x^2-1}\right)^3$.

35. Given $f(x) = 3x^5 - 8x^2 + 7x$, find $f'(-1)$.

36. Given $g(x) = e^x(2x - \sin x)$, find $g'(0)$.

37. If $f(x) = e^{3x}$, find $\ln(f'(2))$.

38. If $f(x) = \ln(3x)$, find the value of $f''(2)$.

39. If $f(x) = 2\tan^{-1}(e^x)$, find the value of $f'(0)$.

40. If $f(x) = \sec(x)$, find the value of $f''(\pi/4)$.

Use the table for the following two questions.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	0	-2	4	1
1	0	3	1	2

41. If $F(x) = f(f(x))$, Find the value of $F'(1)$.

42. If $F(x) = g(f(x))$, Find the value of $F'(1)$.

Use the table for the following three questions.

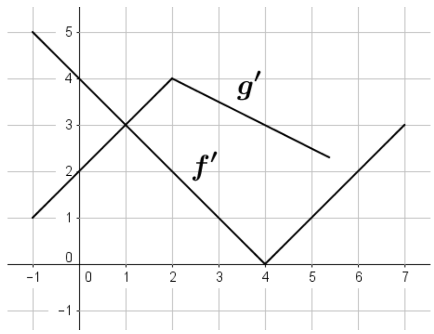
$f(0)$	$f'(0)$	$g(0)$	$g'(0)$
0	-2	4	2

43. If $H(x) = g(f(x)) + x^2 + x$, find $H'(0)$.

44. If $J(x) = 2^{f(x)}$, find $J'(0)$.

45. If $P(x) = e^{f(x)+x^3}$, find $P'(0)$.

The following two questions based on the graph of $y = f'(x)$ and $y = g'(x)$.



46. Given $F(x) = 2f(x) - g(x)$, find the value of $F'(2)$

47. Given $G(x) = f(x) + 2g(x)$, find the value of $G'(0)$

48. Find an **equation of the tangent line** to the curve $y = \sin(\tan x)$ at $x = 0$.

Related Rates

49. Suppose $x^2 + 4y^2 = 17$, where x and y are functions of t. If $\frac{dx}{dt} = 1.5$, find $\frac{dy}{dt}$ if $x = 2$ and $y = 3$.

50. A balloon is rising at a constant speed of 5 ft per sec. A boy is cycling along a straight road at a speed of 15 ft per sec. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 sec later?

51. A ladder 13 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 feet per second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 feet from the wall?

52. Two cars start moving from the same point. One travels south at 40 mph and after one hour, the other travels west at 25 mph. At what rate is the distance between the cars increasing two hours after the second car starts to travel from the point?

53. A street light is mounted at the top of a 15 ft tall pole. A man 6 ft tall walks away from the pole with a speed of 3 ft per sec along a straight path. How fast is the tip of his shadow moving when he is 35 ft from the pole?

54. A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her? Assume the string is taut so that it forms a straight line.

55. If a snowball melts so that its surface area decreases at a rate of $1.5\,cm^2$ per min, find the rate at which the radius decreases when the radius is $12cm$? (Surface Area, $S = 4\pi r^2$)

56. The altitude (height) of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?

57. Each side of a square is increasing at a rate of $3\,\mathtt{in/min}$. At what rate is the area of the square increasing when the side of the square is $5\,\mathtt{in}$?

58. The radius of a circle is increasing at the rate of 5 inches per second. At what rate is the area increasing when the radius is 7 inches?

59. The height of a triangle is increasing at a rate of 3 cm per min while the base of the triangle is increasing at a rate of 5 cm per min. At what rate is the area of the triangle changing when the height is 10 cm and the area is 180 sq. cm?