Calculus I: Test 1 Review Test1: Friday, 2/7/2020

Estimating Limits by Table

1. Guess the value of the following limit by evaluating the function at the given values of x. (Complete the table)

   (a) $\lim_{x \to 3^-} \frac{x}{x-3}$

x	$\frac{x}{x-3}$
2.9	
2.99	
2.999	
2.9999	

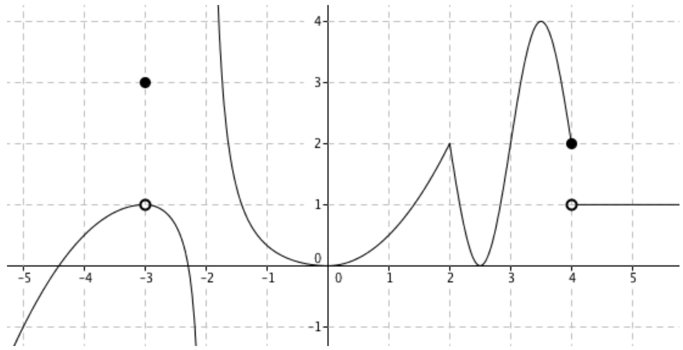
$\lim_{x \to 3^-} \frac{x}{x-3} =$

   (b) $\lim_{x \to 3^-} \frac{x}{3-x}$

x	$\frac{x}{3-x}$
2.9	
2.99	
2.999	
2.9999	

$\lim_{x \to 3^-} \frac{x}{3-x} =$

Limits Graphically



2. Find the following values, if they exist.

   (a) $\lim_{x\to 4^-} f(x) =$

   (b) $\lim_{x\to 4^+} f(x) =$

   (c) $\lim_{x\to 4} f(x) =$

   (d) $f(4) =$

   (e) $\lim_{x\to -3} f(x) =$

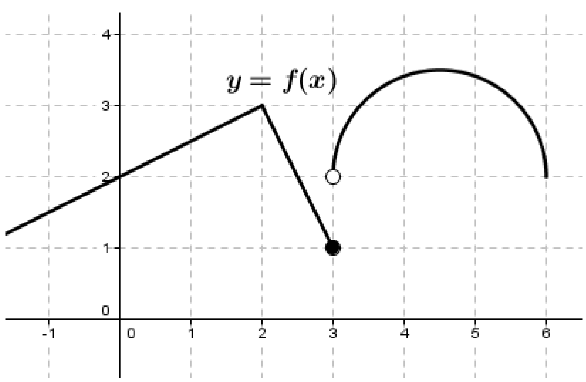
   (f) $\lim_{x\to \infty} f(x) =$

   (g) $\lim_{x\to -2^-} f(x) =$

   (h) $\lim_{x\to -2^+} f(x) =$

   (i) $f(-3) =$

3. Consider the graph of the function $y = f(x)$.



   (a) Evaluate $\lim_{x\to 0} f(x)$

   (b) Evaluate $\lim_{x\to 3^+} f(x)$

   (c) Evaluate $\lim_{x\to 3^-} f(x)$

   (d) Evaluate $\lim_{x\to 3} f(x)$

   (e) Evaluate $f(3)$

   (f) Find the x-values in the interval $(0, 5)$ at which f is NOT continuous and classify them as **removable, jump** or **infinite** discontinuities.

   (g) Find the value(s) of x in the interval $(0, 5)$ at which f is not differentiable.

Limits Algebraically

4. $\lim_{x \to 2^+} \frac{x-2}{|2-x|}$

5. $\lim_{x \to 0} \frac{3}{1-\sin x}$

6. $\lim_{x \to \frac{1}{2}} \left(\cos(\pi x) - e^{x-\frac{1}{2}}\right)$

7. $\lim_{x \to 1/2} \cos\left(\frac{\pi x}{2}\right)$

8. $\lim_{h \to 0} \frac{(2+h)^2 - 4}{h}$

9. $\lim_{x \to 5} \frac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

10. $\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

11. $\lim_{x \to 3} \frac{\frac{1}{x^2 + 2x} - \frac{1}{15}}{x - 3}$

12. $\lim_{x \to 9} \dfrac{x - 9}{3 - \sqrt{x}}$

13. $\lim_{h \to 0} \dfrac{\sqrt{16 + h} - 4}{h}$

14. $\lim_{h \to -4} \dfrac{\sqrt{12 - h} - 4}{h + 4}$

15. $\lim_{x \to 3} \frac{x^3 - 9x}{x^2 - 5x + 6}$

16. $\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$

17. $\lim_{x \to -\infty} \frac{2x^4 + 5x^2 + 2x}{3x^2 - x + 7}$

18. $\lim_{x \to \infty} \frac{x^7 + 3x^5 + 6}{6x^8 + 4x^3 + x^2}$

19. $\lim_{x\to\infty} \dfrac{3x^5 + 5x + 2}{2x^5 - 2x^3 + 1}$

20. $\lim_{x\to\infty} \dfrac{x^4 + 5x^2 - 5}{2x^4 - 2x^3 + 3}$

21. $\lim_{x\to\infty} \dfrac{x^2 - 5x^3 + 10}{2x^2 - 3x + 7}$

22. $\lim_{x\to-\infty} \dfrac{5x^8 + 2}{7x^9 - 3}$

23. $\lim_{x\to\infty} \left(3 + \dfrac{\sin x}{x^2}\right)$

24. $\lim_{x \to 2^-} \frac{x}{x-2}$

25. $\lim_{x \to 3^-} \frac{5}{3-x}$

26. $\lim_{x \to -2^+} \frac{x}{x+2}$

27. $\lim_{x \to -1} \frac{x^2-x-2}{7x+7}$

28. Let $f(x) = \begin{cases} \dfrac{x-4}{x^2-6x+8} & \text{when} \quad x > 3\,, \\ -\dfrac{7}{2+x} & \text{when} \quad x \le 3\,. \end{cases}$

Evaluate the following.

(a) $\lim\limits_{x \to 3^+} f(x)$

(b) $\lim\limits_{x \to 3^-} f(x)$

(c) $\lim\limits_{x \to 3} f(x)$

(d) $\lim\limits_{x \to 4} f(x)$

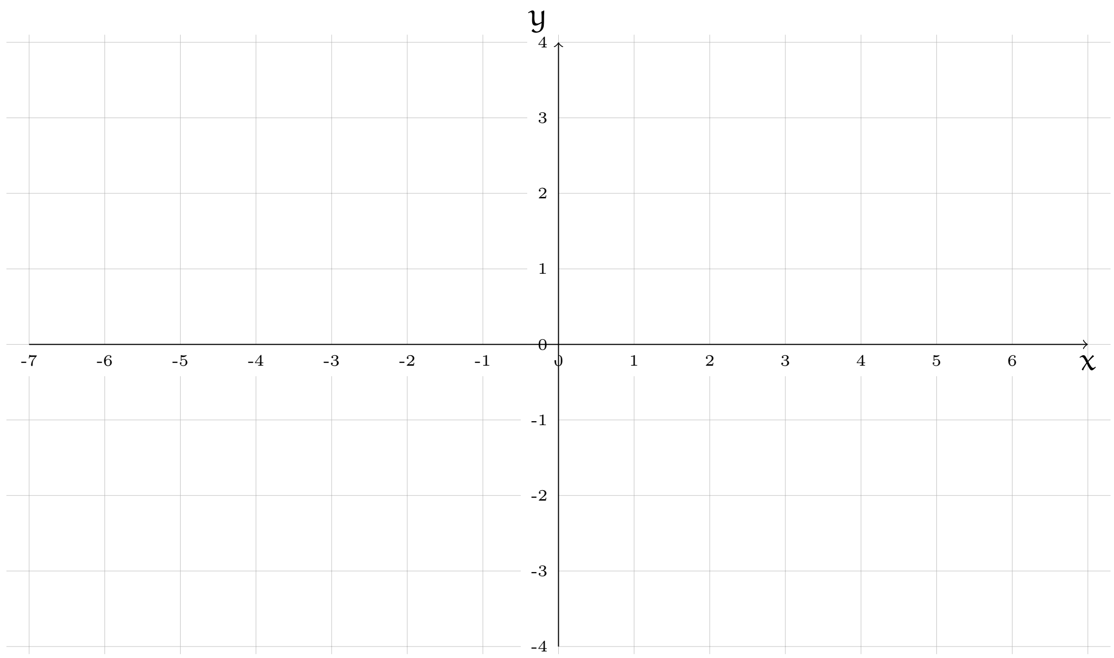
(e) $\lim\limits_{x \to -2^+} f(x)$

(f) $\lim\limits_{x \to \infty} f(x)$

<u>Sketching Graphs</u>

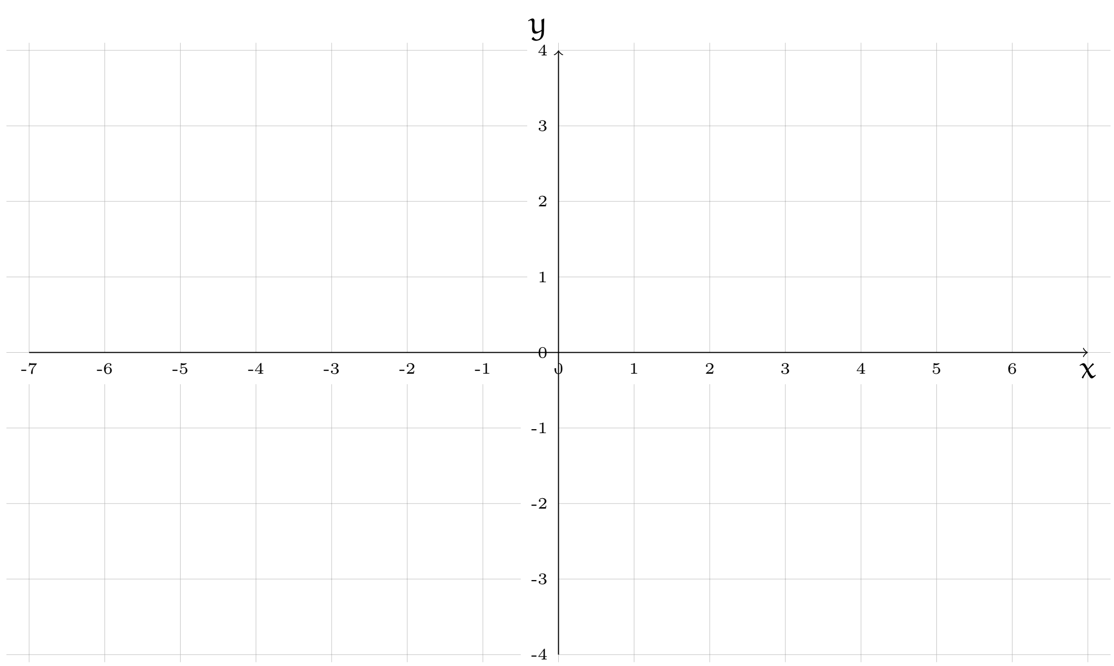
29. Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$f(0) = 1, f(2) = 0 \lim_{x\to 1^-} f(x) = \infty, \lim_{x\to 1^+} f(x) = -\infty, \lim_{x\to\infty} f(x) = 2, \lim_{x\to -\infty} f(x) = 0.$$



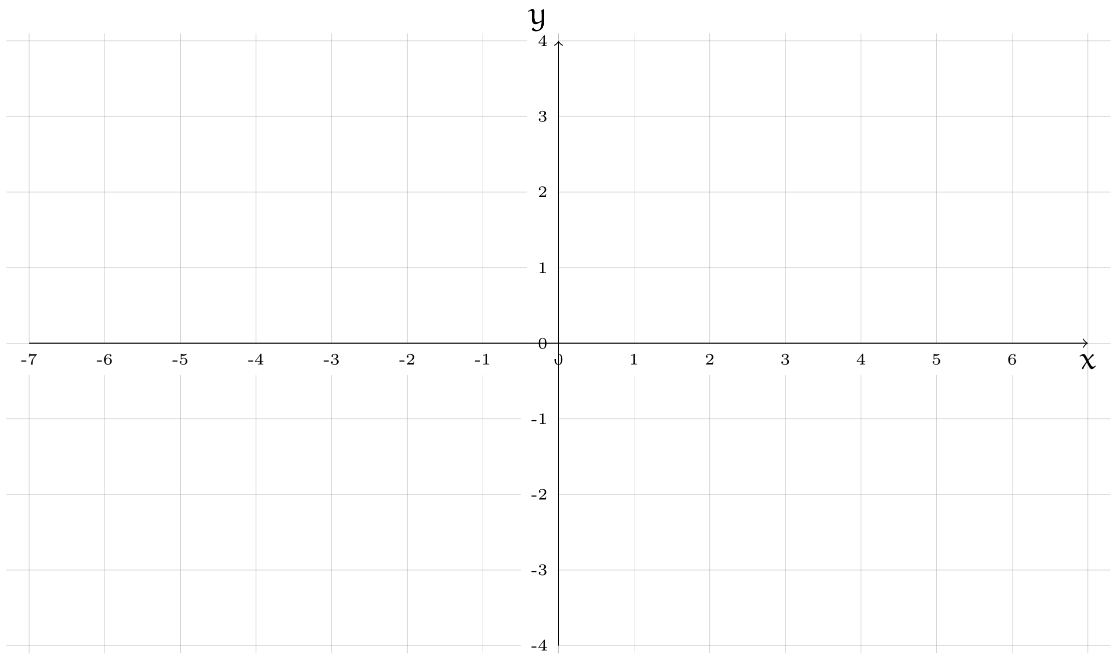
30. Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x\to 1^-} f(x) = \infty, \lim_{x\to 1^+} f(x) = -\infty, \lim_{x\to -\infty} f(x) = -2, f(0) = 0, \lim_{x\to\infty} f(x) = 0$$



31. Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x \to 2} f(x) = -\infty, \ \lim_{x \to -\infty} f(x) = 2, \ \lim_{x \to \infty} f(x) = \infty, \ f(0) = 1, \ f(3) = 0$$



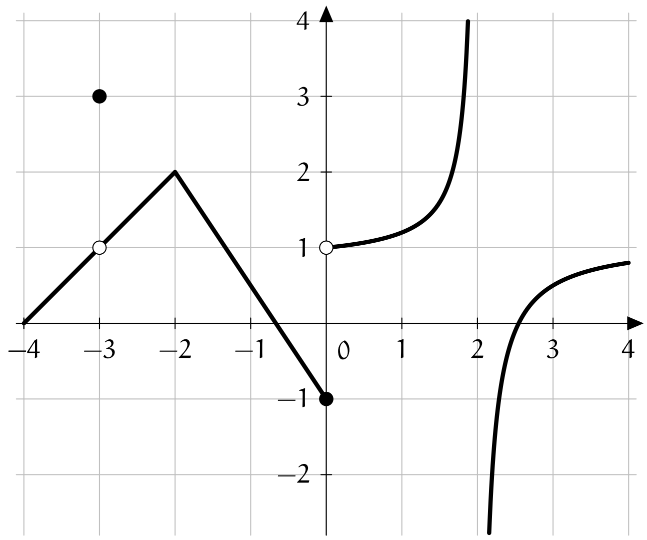
Squeeze Theorem

32. Let $g(x)$ be a function such that $\cos(x) \leq g(x) \leq -\cos(x)+2$ for every x. Find $\lim_{x \to 0} g(x)$.

33. Let $f(x)$ be a function with the property that $5x - 22 \leq f(x) \leq x^3 - 7x - 6$ for every x right of the y-axis. Find $\lim_{x \to 2} f(x)$.

34. If $3 - x^2 \leq f(x) \leq 7 - 4x$ for all x, find the value of $\lim_{x \to 2} f(x)$.

Continuity/Discontinuity



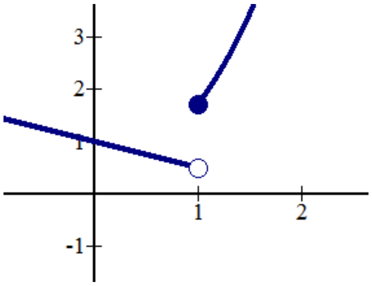
35. Find the x-value (s) in the interval $(-4, 4)$ where f is NOT continuous and classify them as **removable, jump** or **infinite**.

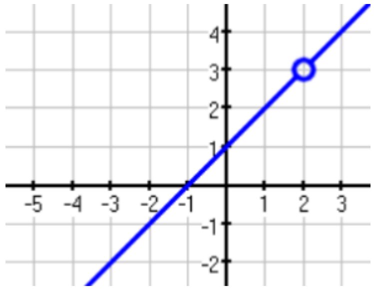
x-value	Discontinuity Type

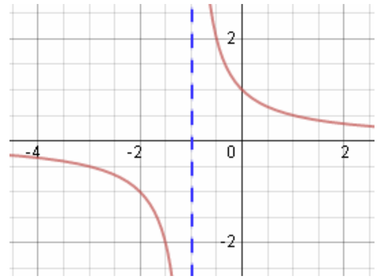
36. Find the value(s) of x in the interval $(-4, 4)$ at which f is not differentiable. **Separate your answers with commas.**

$x =$ ______________________

37. Consider the graphs below. Classify each of their discontinuities as removable (ie. hole), a jump, or infinite (ie. vertical asymptote).







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38. Determine whether the function

$$f(x) = \frac{x^2 - 4}{x - 2}$$

is continuous at $x = 2$. Show your work using the definition of continuity. If f is discontinuous at $x = 2$, state what type of discontinuity (removable, jump or infinite) is this.

39. Is the following function continuous at $x = 2$? Why or why not? Show your work using the definition of continuity.

$$f(x) = \begin{cases} \dfrac{3x - 6}{x - 2}, & x \neq 2, \\ 4, & x = 2. \end{cases}$$

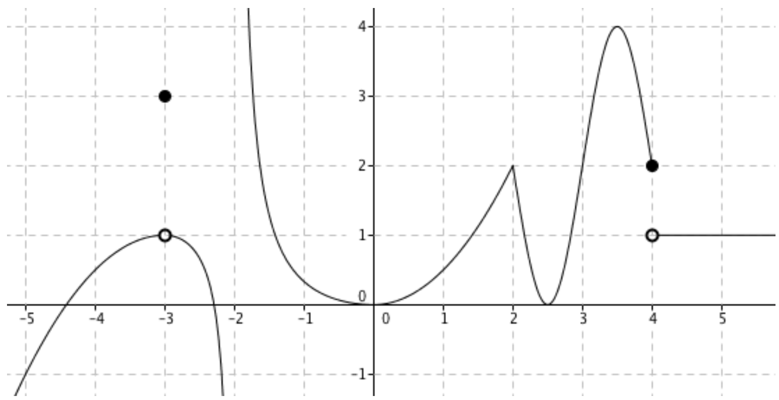
40. Let $f(x)$ be the function given below. Is $f(x)$ continuous on $(-\infty, \infty)$? Show your work using the definition of continuity.

$$f(x) = \begin{cases} x^2 + 3x - 10, & \text{if } x < 3, \\ 9, & \text{if } x = 3, \\ 2x + 2, & \text{if } x > 3. \end{cases}$$

41. Find the value of K that makes the function $f(x) = \begin{cases} Kx^2 + 2x, & x < 2 \\ Kx^3, & x \geq 2 \end{cases}$ continuous on $(-\infty, \infty)$.

42. Consider the function $g(x) = \begin{cases} 2x + 4 + \sin(x - 1) & \text{when } x \leq 1, \\ Cx^2 - 3 & \text{when } x > 1. \end{cases}$ Find the value of C that makes $g(x)$ a continuous function.

43. For what value of the constant k is the function $f(x)$ continuous on $(-\infty, \infty)$, where $f(x) = \begin{cases} x + 5, & x < 1 \\ 2k + x, & x \geq 1. \end{cases}$



44. Is the function graphed above continuous for the following values of x? In the table below, circle YES or NO. If you circle yes, justify your answer using the definition of continuity. If you circle no, state what type of discontinuity (**removable, jump or infinite**) occurs at that value of x.

x value	Continuous?	Reason/Type
-3	YES NO	
-2	YES NO	
2	YES NO	

45. Doe the function graphed above have a derivative at each of the following x values? In the table below, circle YES or NO.

x value	Differentiable?
2	YES NO
4	YES NO
5	YES NO

46. Given that f is continuous at $x = 1$. Which of the following **<u>must</u>** be true?

I. f is differentiable at $x = 1$ II. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$ III. $\lim_{x\to 1} f(x) = f(1)$

Intermediate Value Theorem

47. The function $f(x) = x^2 - 4x$ outputs a function value of 2 for some x between $x = 1$ and $x = 6$. Explain.

48. Use the Intermediate Value Theorem to show that the equation $x^5 + 8x + 4 = 0$ has a solution in the open interval $(-1, 0)$. Explain why the Intermediate Value Theorem applies.

49. Consider the following functions f on the given intervals $[a, b]$. Explain why the Intermediate Value Theorem applies or does not apply on the interval $[a, b]$. If it applies, use it to show that f has a zero in the interval (a, b).

(a) $f(x) = x^5 + 8x + 7$ on the interval $[-1, 0]$.

(b) $f(x) = \dfrac{1}{x - 2}$ on the interval $[0, 4]$.

Derivatives by Definition

50. Find the derivative of the following function using the **definition** of derivative. (Do not use any shortcuts.)

$$f(x) = x^2 + 3.$$

51. Find the derivative of the function using **definition** of derivative. (Do not use any shortcuts.)

$$f(x) = 3x^2 - 5x + 2$$

52. Find the derivative of the function using the **definition** of the derivative.

$$f(x) = \sqrt{x+6}$$

53. Use the definition of the derivative to find $p'(x)$ when $p(x) = \sqrt{x}$.

54. Use the definition of the derivative to find $q'(x)$ when $q(x) = \dfrac{1}{x}$.

Tangent Lines

55. The function $f(x) = x^2 + 5x - \cos(x-2)$ has a derivative of $f'(x) = \sin(x-2) + 2x + 5$.

 (a) Find the instantaneous rate of change of $f(x)$ at $x = 2$.

 (b) Find the equation of the tangent line to $f(x)$ at the point $(2, 13)$.

56. Find the equation of the tangent line to the curve $f(x) = x^2 - 7x$ at $(-1, 8)$ in the slope-intercept form, $y = mx + b$. Given that $f'(x) = 2x - 7$.

57. Find an equation of the line tangent to $f(x) = e^x(2x+1)$ at the point $x = 0$. Given $f'(x) = e^x(2x+3)$.

Average Velocity and Instantaneous Velocity

58. The function $s(t)$ represents the position of an object at time t moving along a line. Suppose $s(1) = 84$ and $s(4) = 144$. Find the average velocity of the object over the interval of time $[1, 4]$.

59. The position function (in feet) of a particle moving in a straight line is given by $s(t) = t^3 + t + 10$, where t is measured in seconds. Find the average velocity (in feet per second) of the particle on the time interval $[1, 3]$.

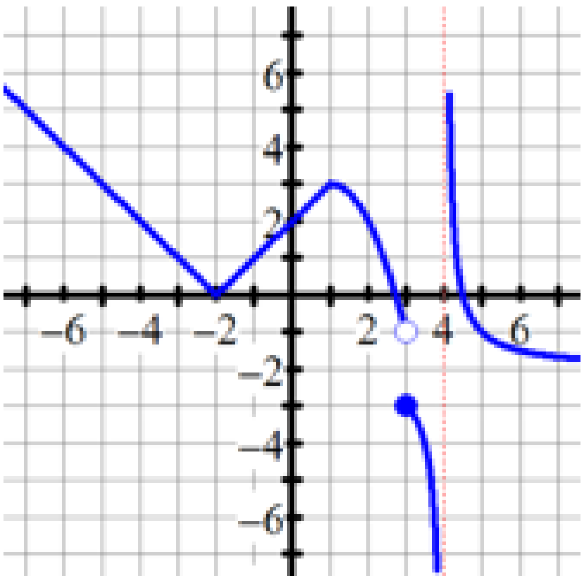
60. The position (in feet) of a particle moving in a straight line is given by $s(t) = t^3 - 10t + 5$, where t is measured in seconds. Find the average velocity (in feet per second) of the particle on the time interval $[1, 3]$.

61. The position (in feet) of a particle moving in a straight line is given by $s(t) = t^3 - 10t + 5$, where t is measured in seconds. Also, it is given that $s'(t) = 3t^2 - 10$. Find the instantaneous velocity (in feet per second) of the particle at $t = 2$.

62. The position function (in feet) of a particle moving in a straight line is given by $s(t) = 2t^3 - 5t + 10$, where t is measured in seconds. Also, it is given that $s'(t) = 6t^2 - 5$. Find the instantaneous velocity (in feet per second) of the particle at $t = 1$.

Differentiability/Nondifferentiability

63. Consider the graph of $g(x)$ given below:



(a) Is $g(x)$ differentiable at $x = 3$? Explain.

(b) Is $g(x)$ differentiable at $x = 1$? Explain.

(c) Is $g(x)$ differentiable at $x = 5$? Explain.

64. The graph of f is given below. State the numbers at which f is not differentiable.