Formula: **Linearization** of \( f \) at \( a \) is given by
\[ L(x) = f(a) + f'(a)(x - a) \]

1. Find the linearization \( L(x) \). (a) \( f(x) = \ln x \), \( a = 1 \) \hspace{1cm} (b) \( f(x) = \sqrt{x + 3} \), \( a = 1 \).

(a) To use the above mentioned formula we find
\[ f(a) = f(1) = \ln 1 = 0, \text{ derivative of } f, \ f'(x) = \frac{1}{x}, \text{ and evaluate it at } a, \ f'(a) = f'(1) = \frac{1}{1} = 1. \]

Now using the formula we get,
\[ L(x) = f(a) + f'(a)(x - a) = 0 + 1(x - 1) = x - 1. \]

(b) To use the above mentioned formula we find
\[ f(a) = f(1) = \sqrt{1 + 3} = 2, \text{ derivative of } f, \ f'(x) = \frac{1}{2}(x + 3)^{-1/2}, \text{ and evaluate it at } a, \]
\[ f'(a) = f'(1) = \frac{1}{2}(4)^{-1/2} = \frac{1}{2} \cdot \frac{1}{\sqrt{4}} = \frac{1}{4}. \]

Now using the formula we get,
\[ L(x) = f(a) + f'(a)(x - a) = 2 + \frac{1}{4}(x - 1) = 2 + \frac{1}{4}x - \frac{1}{4} = \frac{1}{4}x + \frac{7}{4}. \]

2. Use a linear approximation to estimate the given number.

(a) \( (4.01)^{1/2} \) \hspace{1cm} (b) \( e^{-0.02} \) \hspace{1cm} (c) \( \sqrt[3]{27.1} \).

(a) Assume \( f(x) = x^{1/2} \), take \( a = 4 \). Then, \( f'(x) = \frac{1}{2}x^{-1/2} \).

So we have slope, \( f'(a) = f'(4) = \frac{1}{2}4^{-1/2} = \frac{1}{4} \), and point \( (a, f(a)) = (4, 4^{1/2}) = (4, 2) \).

From the linearization formula, \( L(x) = f(a) + f'(a)(x - a) = 2 + \frac{1}{4}(x - 4) \).

Thus, \( (4.01)^{1/2} \approx 2 + \frac{1}{4}(4.01 - 4) = 2 + \frac{1}{4}(.01) = 2.0025 \).

(b) Assume \( f(x) = e^x \), take \( a = 0 \). Then, \( f'(x) = e^x \).

So we have slope, \( f'(a) = f'(0) = 1 \), and point \( (a, f(a)) = (0, e^0) = (0, 1) \).

From the linearization formula, \( L(x) = f(a) + f'(a)(x - a) = 1 + 1(x - 0) \).

Thus, \( e^{-0.02} \approx 1 + 1(-0.02 - 0) = 1 - 0.02 = 0.98 \).

(c) We can rewrite the number as \( (27.1)^{1/3} \). Assume \( f(x) = x^{1/3} \), take \( a = 27 \). Then, \( f'(x) = \frac{1}{3}x^{-2/3} \).

So we have slope, \( f'(a) = f'(27) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27} \), and point \( (a, f(a)) = (27, (27)^{1/3}) = (27, 3) \).

From the linearization formula, \( L(x) = f(a) + f'(a)(x - a) = 3 + \frac{1}{27}(x - 27) \).

Thus, \( (27.1)^{1/3} \approx 3 + \frac{1}{27}(27.1 - 27) = 3 + \frac{1}{27}(.1) = 3.0037 \).

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