1. Find the following indefinite integrals.

6. (a) \[ \int (10x^4 - 2x + 1) \, dx = \frac{10x^5}{5} - \frac{2x^2}{2} + x + C = 2x^5 - x^2 + x + C \]

Using \[ \int x^n \, dx = \frac{x^{n+1}}{n+1} + C \]

6. (b) \[ \int (e^{2x} + 2\sqrt{x}) \, dx = \int e^{2x} + 2x^{1/2} \, dx = \frac{1}{2} e^{2x} + \frac{2x^{3/2}}{3} + C \]

Note: \[ \frac{d}{dx} e^{2x} = 2e^{2x} \]

So \[ \frac{d}{dx} \left( \frac{1}{2} e^{2x} \right) = e^{2x} \]

6. (c) \[ \int 4\sin(2\theta) - \sec^2(3\theta) \, d\theta = 4 \left( -\frac{1}{2} \cos(2\theta) \right) - \frac{1}{3} \tan(3\theta) + C \]

Note: \[ \frac{d}{dx} \tan(3x) = 3\sec^2(3x) \]

So \[ \frac{d}{dx} \left[ \frac{1}{3} \tan(3x) \right] = \sec^2(3x) \]

6. (d) \[ \int \frac{1 + x - x^2}{x^2} \, dx = \int \frac{1}{x^2} + \frac{x}{x^2} - \frac{x^2}{x^2} \, dx \]

\[ = \int x^{-2} + \frac{1}{x} - 1 \, dx = \frac{x^{-1}}{-1} + |x| - x + C = \frac{1}{x} + |x| - x + C \]
2. Find the absolute maximum and absolute minimum values of the function on the given interval.

\[ f(x) = 2x^3 - 24x, \quad [-3, 1] \]

\[ f'(x) = 6x^2 - 24 = 0 \]

\[ 6x^2 = 24 \quad \rightarrow \quad x^2 = 4 \quad \rightarrow \quad x = \pm 2 \]

|x|=2 \quad \text{is not in} \quad [-3, 1]

\[ \text{Max}/\text{Min} \quad \text{occur at} \quad x = -3, -2, 1 \]

\[ f(-3) = 2(-27) - 24(-3) = -54 + 72 = 18 \]

\[ f(-2) = 2(-8) - 24(-2) = -16 + 48 = 32 \]

\[ f(1) = 2(1) - 24(1) = 2 - 24 = -22 \]

\[ \text{Max is} \quad 32 \quad \text{at} \quad x = -2 \]

\[ \text{Min is} \quad -22 \quad \text{at} \quad x = 1 \]

3. The function \( f(x) = 3x^2 + 2x + 5 \) is continuous on \([-1, 1]\) and differentiable on \((-1, 1))\) being a polynomial. Find all numbers \( c \) in \((-1, 1)\) satisfying the conclusion of the Mean Value Theorem for the function \( f \).

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

\[ f(1) = 3 + 2 + 5 = 10 \]

\[ f(-1) = 3 - 2 + 5 = 6 \]

\[ \frac{f(b) - f(a)}{b - a} = \frac{10 - b}{1 - (-1)} \quad = \frac{4}{2} = 2 \]

\[ f'(c) = 6c + 2 \]

\[ \frac{6c + 2}{b - \sqrt{a}} = 2 \]

\[ 6x + 2 = 2 \]

\[ 6x = 0 \]

\[ x = 0 \]
4. The derivatives of $f$ are given by $f'(x) = \frac{-4x}{(x^2 - 1)^2}$, $f''(x) = \frac{12x^2 + 4}{(x^2 - 1)^3}$. Use this to answer the following questions about $f$.

(a) Find the intervals on which $f$ is increasing or decreasing.

Test $f'$ at $x = -2, \frac{1}{2}, 1, 2$

- $f'(-2) > 0$ (note the denominator is positive)
- $f'\left(\frac{1}{2}\right) > 0$
- $f'(1) < 0$
- $f'(2) < 0$

Increasing on $(-\infty, -1)$ and $(-1, 0)$.
Decreasing on $(0, 1)$ and $(1, \infty)$.

(b) Find the $x$-values at which $f$ has a local maximum or minimum. Identify which is which.

Local max at $x = 0$ from First Derivative Test.

(c) Find the intervals on which $f$ is concave up or down.

Numerical of $f''$ is always positive.

Test $f''$ at $-2, 0, 2$

- $f''(-2) > 0$
- $f''(0) < 0$
- $f''(2) > 0$

(d) Find the $x$-coordinates of inflection point(s) of $f$, if any.

There are inflection points at $x = \pm 1$ if $f(x)$ is defined at those points. If it is not, then they are not considered inflection points.
5. Use a linear approximation to approximate the value of $\sqrt{16.05}$. (Must define a function and use its derivative to get answer)

$$f(x) = \sqrt{x}, \quad x_0 = 16, \quad f(x_0) = \sqrt{16} = 4$$

$$f'(x) = \frac{1}{2\sqrt{x}}, \quad f'(x_0) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$$

$$L(x) = f(x_0) + f'(x_0)(x-x_0)$$

$$= 4 + \frac{1}{8}(x-16)$$

$$\sqrt{16.05} \approx L(16.05) = 4 + \frac{1}{8}(16.05-16) = 4 + \frac{1}{8}(0.05)$$

$$= 4 + \frac{1}{160}$$

6. A farmer with 2400 feet of fencing wants to enclose a rectangular area that borders a straight river, and then divide it into three pens with fencing perpendicular to the river. Find the dimensions of the rectangle that yield the largest possible total area of the four pens.

![Diagram of a rectangular area with fencing]

Maximize $A = xy$

Subject to $x + 4y = 2400$ (material constraint)

Solve constraint for $y$: $y = \frac{2400-x}{4}$

Substitute into objective

$$A = x\left(600 - \frac{x}{4}\right) = 600x - \frac{x^2}{4}$$

parabolic, opens down, has a max when $A' = 0$

$$A' = 600 - \frac{2x}{4} = 0 \rightarrow 600 = \frac{1}{2}x \rightarrow x = 1200$$

$$y = \frac{2400 - 1200}{4} = 300$$
7. Evaluate the following limit, if it exists.

\[ \lim_{{x \to -1}} \frac{x^5 + 1}{x^4 - 1} = \frac{0}{0} \]

\[ = \lim_{{x \to -1}} \frac{5x^4}{4x^3} = \frac{5}{-4} = \left( -\frac{5}{4} \right) \]

(b) \[ \lim_{{x \to \infty}} x \tan\left(\frac{2}{x}\right) = \infty \cdot 0 = \lim_{{x \to \infty}} \frac{\tan\left(\frac{2}{x}\right)}{\frac{1}{x}} = \frac{0}{0} \]

\[ = \lim_{{x \to \infty}} \frac{\sec^2\left(\frac{2}{x}\right) \cdot \left(\frac{-2}{x^2}\right)}{\left(\frac{-1}{x^2}\right)} = \lim_{{x \to \infty}} 2 \sec^2\left(\frac{2}{x}\right) = 2 \sec^2(0) = 2. \]

8. A large tank is filled with water when an outflow valve is opened at \( t = 0 \). Water flows out at a rate, in gal/min, given by

\[ Q'(t) = 0.1(100 - t^2), \quad 0 \leq t \leq 10. \]

How much water flows out of the tank in 10 minutes?

Answer: \( Q(10) - Q(0) \). Need to find \( Q(t) \)

Since \( Q(t) = \int 0.1(100 - t^2) \, dt \), \( Q(t) = \int Q'(t) \, dt \)

\[ = \int 0.1(100 - t^2) \, dt = 0.1 \left( 100t - \frac{t^3}{3} \right) + C \]

So \( Q(10) - Q(0) = Q(10) \) (since \( Q(0) = 0 \))

\[ = 0.1 \left( 1000 - \frac{10000}{3} \right) = \frac{2}{3} (100) = \frac{200}{3} \text{ gallons} \]