Please read each question carefully to ensure that you are actually answering it. **Show all work.** All numerical answers should be left in exact form unless otherwise specified.

1. Consider the function \( f(x) = 2x^3 - 3x^2 - 12x + 5 \). Answer the following using **calculus**. (Answer produced from graphing calculator receives no credit.)

   (a) Find the intervals on which \( f \) is increasing or decreasing. Answer in the space provided.

   \[
   f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)
   \]

   \[\begin{align*}
   &\text{Right point: } x = -2, b, 1 \quad \text{where } f'(-2) > 0, f'(b) < 0, f'(1) > 0 \\
   &\text{Left point: } x = 1, -2 \\
   &f \text{ is increasing on } (-\infty, -2) \text{ and } (1, +\infty) \\
   &f \text{ is decreasing on } (-2, 1).
   \end{align*}\]

   (b) Find the \( x \)-values where \( f \) attains its local maximum and minimum values.

   See diagram above and use the **First Derivative Test**

   \[
   f \text{ has local maximum at } x = -1 \\
   f \text{ has local minimum at } x = 2
   \]

2. Consider the function \( f(x) = 2x^3 - 3x^2 \). Answer the following using **calculus**. (Answer produced from graphing calculator receives no credit.)

   (a) Find the intervals on which \( f \) is concave up or concave down.

   \[
   f'(x) = 6x^2 - 6x \quad f''(x) = 12x - 6 = 0 \implies x = \frac{1}{2}
   \]

   \[\begin{align*}
   &\text{Right point: } x = 0, 1 \\
   &f''(0) = -6 < 0 \\
   &f''(1) = 6 > 0
   \end{align*}\]

   \[
   f \text{ is concave up on } (-\infty, 0) \text{ and } (0, 1) \\
   f \text{ is concave down on } (1, +\infty)
   \]

   (b) Find the \( x \)-coordinate(s) of inflection point(s) of \( f \).

   See diagram above

   \[
   f \text{ has inflection point(s) at } x = \frac{1}{2}
   \]
3. Find the absolute maximum and absolute minimum values of \( f \) on the given interval.

\[
f(x) = x^3 - 6x^2 + 9x + 2, \quad [-2, 2]
\]

\[
f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)
\]

\[
= 3(x - 3)(x - 1) = 0
\]

\[
x = 1, 3 \quad \text{shown } \quad x = 3, \text{ not in interval } [-2, 2]
\]

Compute:

\[
f(-2) = -8 - 24 - 18 + 2 = -48
\]

\[
f(1) = 1 - 6 + 9 + 2 = 6
\]

\[
f(2) = 8 - 24 + 18 + 2 = 4
\]

Max is 6 at \( x = 1 \)

Min is -48 at \( x = -2 \)

4. Evaluate the following limits.

(a) \( \lim_{x \to \infty} 5xe^{-x} = \infty \cdot 0 \)

\[
= \lim_{x \to \infty} \frac{5x}{e^x} = \frac{\infty}{\infty}
\]

\[
= \lim_{x \to \infty} \frac{5}{e^x} = \frac{5}{\infty} = 0.
\]

(b) \( \lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1 - 1}{0} = \frac{0}{0} \)

\[
\left. \frac{\sin x}{2x} \right|_0
\]

\[
= \lim_{x \to 0} \frac{\sin x}{2x} = \frac{0}{0}
\]

\[
\left. \frac{\cos x}{2} \right|_0
\]

\[
= \left. \frac{\cos 0}{2} \right|_0 = \frac{1}{2}
\]
5. Sketch the graph of a function $f$ that satisfies all of the given conditions.

1. $f(-1) = 4$, $f(1) = -2$, and $f(3) = 1$.
2. $f'(-1) = f'(1) = f'(3) = 0$.
3. $f'(x) > 0$ if $x < -1$ or $1 < x < 3$.
4. $f'(x) < 0$ if $-1 < x < 1$ or $x > 3$.
5. $f''(x) > 0$ if $0 < x < 2$.
6. $f''(x) < 0$ if $x < 0$ or $x > 2$.

Circle the correct answer. You do not need to show your work. (No partial credit)

3. \[
\int \frac{1}{1 + x^2} \, dx = \tan^{-1}(x) + C
\]

(a) $\ln(1 + x^2) + C$   (b) $\tan^{-1} x + C$   (c) $\frac{x}{x + \frac{1}{3}x^3} + C$   (d) $\cos^{-1} x + C$   (e) $x - \frac{1}{x} + C$
7. The Mean Value Theorem guarantees the existence of a special number $c$ in the interval $(0,16)$ for the function $f(x) = \sqrt{x}$. Find the number $c$.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\frac{f(b) - f(a)}{b - a} = \frac{\sqrt{16} - \sqrt{0}}{16 - 0} = \frac{4}{16} = \frac{1}{4}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

Since $f'(x) = \frac{1}{2\sqrt{x}}$, we have:

$$\frac{1}{2\sqrt{x}} = \frac{1}{4} \quad \Rightarrow \quad 4 = 2\sqrt{x}$$

$$2 = \sqrt{x}$$

$$x = 4$$

8. Use a linear approximation to approximate the value of $e^{0.2}$. (Must define a function and use its derivative to get answer)

$$f(x) = e^x, \quad x_0 = 0, \quad f(x_0) = f(0) = e^0 = 1$$

$$f'(x) = e^x, \quad \text{so} \quad f'(x_0) = f'(0) = e^0 = 1$$

$$L(x) = f(x_0) + f'(x_0) (x-x_0)$$

$$= 1 + 1 \ (x-0)$$

$$L(x) = 1 + x$$

Now $e^{0.2} \approx L(0.2) = 1 + 0.2 = 1.2$. 
9. Evaluate the following integrals.

4. (a) \( \int x^4 - 3x^2 + 1 \, dx \)
   \[= \frac{x^5}{5} - \frac{3x^3}{3} + x + C\]
   \[= \frac{x^5}{5} - x^3 + x + C\]

   (Using: \( \int x^n \, dx = \frac{x^{n+1}}{n+1} + C \))

5. (b) \( \int x \left( 1 - \frac{1}{\sqrt{x}} \right) \, dx \)
   \[= \int x - \frac{x}{\sqrt{x}} \, dx = \int x - x^{\frac{1}{2}} \, dx\]
   \[= \frac{x^2}{2} - \frac{2}{3} x^{\frac{3}{2}} + C\]

4. (c) \( \int e^x + \sin x + \frac{1}{\cos^2 x} \, dx \)
   \[= \int e^x + \sin x + \sec^2 x \, dx\]
   \[= e^x - \cos x + \tan x + C\]
A box with an open top is to be constructed from a square piece of cardboard, 6 feet wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

\[ V = (6-2x)^2 \cdot x = (36 - 24x + 4x^2) \cdot x \]

\[ V = 4x^3 - 24x^2 + 36x, \quad 0 \leq x \leq 3 \]

\[ V' = 12x^2 - 48x + 36 = 12(x^2 - 4x + 3) \]

\[ = 12(x-3)(x-1) = 0 \]

\[ x = 1 \text{ or } 3 \]

Max occurs at \( x = 0, 1 \text{ or } 3 \)

\[ V(0) = 0, \quad V(1) = (6-2)^2 \cdot 1 = 16 \]

and \( V(3) = 0 \)

Max value is \( 16 (ft)^3 \) when \( x = 1 \).