

Name Solutions

Score _____

Show all your work to receive credit. All answers must be left in exact and simplified form unless otherwise stated.

1. Perform the indicated conversion.

a) (5 points) Convert 230° to radians. Express your answer as a multiple of π .

$$230^\circ = 230^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = \frac{23\pi}{18} \text{ rad}$$

b) (5 points) Convert $\frac{8\pi}{9}$ radians to degrees.

$$\frac{8\pi}{9} \text{ rad} = \frac{8\pi}{9} \cdot \frac{180^\circ}{\pi \text{ rad}} = \frac{8 \cdot 180^\circ}{9} = 8 \cdot 20^\circ = 160^\circ$$

2. (8 points) Let s denote the length of the arc of a circle of radius r subtended by the central angle θ . Given $r = 6$ feet, $\theta = 70^\circ$, find s with the correct units. Round your answer to two decimal places.

convert θ to radians: $70^\circ = 70^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = \frac{7\pi}{18} \text{ rad}$

use $s = r\theta = 6 \cdot \frac{7\pi}{18} = \frac{7\pi}{3} \text{ ft}$

3. (6 points) The point $\left(-\frac{2}{5}, \frac{\sqrt{21}}{5}\right)$ is a point on the unit circle that corresponds to a real number t . Find the exact values of the six trigonometric functions of t .

$$\sin t = \frac{\sqrt{21}}{5}$$

$$\csc t = \frac{5}{\sqrt{21}}$$

$$\cos t = -\frac{2}{5}$$

$$\sec t = -\frac{5}{2}$$

$$\tan t = -\frac{\sqrt{21}}{2}$$

$$\cot t = -\frac{2}{\sqrt{21}}$$

4. (8 points) A water sprinkler sprays water over a distance of 20 feet while rotating through an angle of 170° . What area of lawn receives water? Round your answer to two decimal places.

convert 170° to radians

$$170^\circ = 170^\circ \cdot \frac{\pi}{180} = \frac{17}{18} \pi \text{ rad}$$

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} (20)^2 \left(\frac{17\pi}{18} \right) = \frac{1}{2} \cdot 400 \cdot \frac{17\pi}{18}$$

$$= \frac{1700}{9} \pi \text{ ft}^2$$

$$\approx 593.41 \text{ ft}^2$$

5. Find the **exact value** of each expression. You do not have to rationalize any denominators for this problem. **Show your work.**

a) (5 points) $\csc 30^\circ \tan 60^\circ$

$$\csc 30^\circ = \frac{1}{\sin(30^\circ)} = \frac{1}{(\frac{1}{2})} = 2$$

$$\tan 60^\circ = \frac{(\frac{\sqrt{3}}{2})}{(\frac{1}{2})} = \sqrt{3}$$

$$\csc 30^\circ \cdot \tan 60^\circ = 2 \cdot \sqrt{3}$$

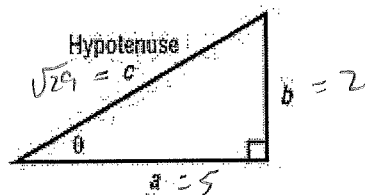
b) (5 points) $8 \sin \frac{\pi}{4} - 4 \cot \frac{\pi}{3}$

$$\cot \left(\frac{\pi}{3} \right) = \frac{\cos(\frac{\pi}{3})}{\sin(\frac{\pi}{3})} = \frac{(\frac{1}{2})}{(\frac{\sqrt{3}}{2})} = \frac{1}{\sqrt{3}}$$

$$= 8 \left(\frac{\sqrt{2}}{2} \right) - 4 \left(\frac{1}{\sqrt{3}} \right)$$

$$= 4\sqrt{2} - \frac{4}{\sqrt{3}}$$

6. (13 points) Find the **exact value** of the six trigonometric functions of the angle θ in the right triangle, where hypotenuse $c = \sqrt{29}$ and side $b = 2$.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 2^2 &= (\sqrt{29})^2 \Rightarrow a^2 + 4 = 29 \\ &\Rightarrow a^2 = 25 \\ &\Rightarrow a = \pm 5 \\ &\Rightarrow \boxed{a = 5} \end{aligned}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{29}}$$

$$\csc \theta = \frac{\sqrt{29}}{2}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{\sqrt{29}}$$

$$\sec \theta = \frac{\sqrt{29}}{5}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{5}$$

$$\cot \theta = \frac{5}{2}$$

7. (12 points) Find the **exact value** of the remaining five trigonometric functions of θ . Draw a picture.

$$\cos \theta = -\frac{5}{8}, \quad \pi < \theta < \frac{3\pi}{2} \quad \text{Quad III} \quad (x < 0, y < 0)$$

$$\sin \theta = -\frac{\sqrt{39}}{8} \quad \csc \theta = -\frac{8}{\sqrt{39}}$$

$$\cos \theta = -\frac{5}{8} \quad \sec \theta = -\frac{8}{5}$$

$$\tan \theta = \frac{\sqrt{39}}{5} \quad \cot \theta = \frac{5}{\sqrt{39}}$$

$$\cos \theta = -\frac{5}{8} \Rightarrow x = -5 \text{ and } r = 8$$

to find y , use $x^2 + y^2 = r^2$

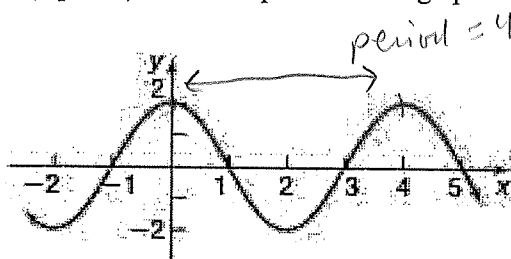
$$(-5)^2 + y^2 = 8^2$$

$$25 + y^2 = 64$$

$$y^2 = 64 - 25 = 39$$

$$y = \pm \sqrt{39} \Rightarrow y = -\sqrt{39} \quad (\text{Quad III})$$

8. (7 points) Find an equation of the graph.

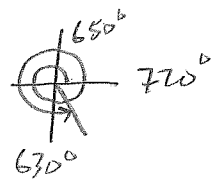


$$\cos(0) = 1 \quad \text{so cosine curve, } y = A \cos(\omega x)$$

$$\text{amp} = 2$$

$$\text{period} = 4 = \frac{2\pi}{\omega} \Rightarrow 4\omega = 2\pi \Rightarrow \omega = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\text{so } \boxed{y = 2 \cos\left(\frac{\pi}{2}x\right)}$$



Circle the correct answer. **No partial credit.**

9. (4 points) Which one of the following describes the location of 650° ?

- (a) Quadrant I (b) Quadrant II (c) Quadrant III (d) Quadrant IV (e) x-axis (f) y-axis

10. (4 points) The amplitude of $y = -\frac{4}{5} \sin\left(\frac{7}{6}x\right)$ is

- (a) $-\frac{4}{5}$ (b) $\frac{4}{5}$ (c) $\frac{5}{4}$ (d) $\frac{6}{7}$ (e) $\frac{7}{6}$

11. (4 points) The period of $y = -\frac{4}{5} \cos\left(\frac{7}{6}x\right)$ is

- (a) 2π (b) $-\frac{4}{5}$ (c) $\frac{7}{6}$ (d) $\frac{12}{7}\pi$ (e) $\frac{7}{6}\pi$

$$\frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{7}{6}\right)} = 2\pi \cdot \frac{6}{7} = \frac{12\pi}{7}$$

12. (4 points) Which Quadrant does the angle θ lie in if $\cos \theta > 0$ and $\tan \theta > 0$?

- (a) Quadrant I (b) Quadrant II (c) Quadrant III (d) Quadrant IV (e) x-axis (f) y-axis

$$x > 0 \quad \frac{y}{x} > 0 \Rightarrow y > 0 \text{ (since } x > 0)$$

13. (4 points) Find the exact value of $\cos 55^\circ \sec 55^\circ$.

- (a) 1 (b) 0 (c) -1 (d) undefined (e) None of the above

$$\sec 55^\circ = \frac{1}{\cos 55^\circ}$$

14. Circle the correct response. **No Partial Credit.**

(a) (2 points) The cosine function is even. That is $\cos(-\theta) = \cos \theta$. TRUE or FALSE

(b) (2 points) The range of $y = \sin x$ is; $(-\infty, \infty)$. TRUE or FALSE

(c) (2 points) The domain of $y = \tan x$ is; $(-\infty, \infty)$. TRUE or FALSE

$\frac{\pi}{2}$ is not in the domain of $\tan x$

it is $[-1, 1]$