

Quiz 9

$A = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$ find the non-zero eigenvalue and its eigenspace

$$\begin{aligned} \det(\lambda I - A) &= \det \begin{pmatrix} \lambda - 3 & -1 \\ -6 & \lambda - 2 \end{pmatrix} = (\lambda - 3)(\lambda - 2) - 6 \\ &= (\lambda^2 - 5\lambda + 6) - 6 = \lambda^2 - 5\lambda = \lambda(\lambda - 5) = 0 \\ \lambda &= 0, \quad \underline{\lambda = 5} \text{ eigenvalues} \end{aligned}$$

$$E_5: \left[\begin{array}{cc|c} 2 & -1 & 0 \\ -6 & 3 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$
$$x_2 = t, \quad x_1 = \frac{1}{2}t$$

$$\vec{x} = t \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} \sim t \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$E_5 = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

$$\dim E_5 = 1.$$