Fall 2015 | Math 344 Quiz 8 | Name: Solutions

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 1 & -1 & 1 & -1 & 0 \\ 2 & 0 & 1 & 2 & 1 \\ 0 & -1 & -3 & 3 & 0 \end{bmatrix}$$

1. Find the RREF of A.

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 1 & -1 & 1 & -1 & 0 \\ 2 & 0 & 1 & 2 & 1 \\ 0 & -1 & -3 & 3 & 0 \end{bmatrix} \xrightarrow[\substack{R_3 - 2R_1 \\ -R_4}]{R_2 - R_1} \begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 0 & -3 & -2 & -1 & -1 \\ 0 & -4 & -5 & 2 & -1 \\ 0 & 1 & 3 & -3 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_4} \begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 0 & 1 & 3 & -3 & 0 \\ 0 & -4 & -5 & 2 & -1 \\ 0 & -3 & -2 & -1 & -1 \end{bmatrix} \xrightarrow[R_4 + 3R_2]{R_3 + 4R_2} \begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 0 & 1 & 3 & -3 & 0 \\ 0 & 0 & 7 & -10 & -1 \\ 0 & 0 & 7 & -10 & -1 \end{bmatrix}$$

$$\xrightarrow[\substack{\text{and } \frac{1}{7}R_3 \\ \text{after}}]{\substack{R_1 - 2R_2 \\ R_4 - R_3}} \begin{bmatrix} 1 & 0 & -3 & 6 & 1 \\ 0 & 1 & 3 & -3 & 0 \\ 0 & 0 & 1 & -\frac{10}{7} & -\frac{1}{7} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_2 - 3R_3]{R_1 + 3R_3} \begin{bmatrix} 1 & 0 & 0 & \frac{12}{7} & \frac{4}{7} \\ 0 & 1 & 0 & \frac{9}{7} & \frac{3}{7} \\ 0 & 0 & 1 & -\frac{10}{7} & -\frac{1}{7} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Find a basis for Null(A). $A\vec{x} = \vec{0}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$

$x_4 = s$

$x_5 = t$

$x_3 = \frac{10}{7}s + \frac{1}{7}t$

$x_2 = -\frac{9}{7}s - \frac{3}{7}t$

$x_1 = -\frac{12}{7}s - \frac{4}{7}t$

$$\vec{x} = s\underbrace{\begin{bmatrix} -\frac{12}{7} \\ -\frac{9}{7} \\ \frac{10}{7} \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -\frac{4}{7} \\ -\frac{3}{7} \\ \frac{1}{7} \\ 0 \\ 1 \end{bmatrix}}_{\text{basis for Null}(A)}$$

3. Find a basis for Row(A).

$\{(7,0,0,12,4),\ (0,7,0,9,3),\ (0,0,7,-10,-1)\}$

(I multiplied the non-zero rows of RREF by 7).

4. Find a basis for Col(A). Columns 1, 2 and 3 of A

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ -3 \end{bmatrix} \right\}$$

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5. Verify that dim(Row A) = dim(Col A) and that Rank A + Nullity A = # Col's of A.

dim (Row A) = 3 ; dim (Col (A)) = 3

Rank A = dim (Row A) = 3

Nullity A = dim (Null A) = 2

$\Rightarrow 3+2 = 5 = \#$ Col's of A.

6. Find a basis for the subspace of P_2 given by $W = \{at^2 + bt + c \mid a + b + c = 0\}$.

$c = -a-b$

$W = \{at^2 + bt + (-a-b)\}$

$= \{a(t^2-1) + b(t-1)\}$

basis is $\{t^2-1,\ t-1\}$