Fall 2015 Math 344 Quiz 7 Name: Solutions

Suppose that $\{v, w\}$ is a linearly independent set in a vector space V. Prove that $\{v - w, 3v + 2w\}$ is a linearly independent set as well.

Since $\{v, w\}$ is a linearly independent set,

if $av + bw = \vec{0}$ then $a = 0$ and $b = 0$.

Now consider the set $\{v - w, 3v + 2w\}$.

If $c_1(v - w) + c_2(3v + 2w) = \vec{0}$ $\quad (*)$

find c_1, c_2.

Regrouping the left hand side:

$$(c_1 + 3c_2)v + (-c_1 + 2c_2)w = \vec{0}$$

By Lin ind of $\{v, w\}$, we have

$$c_1 + 3c_2 = 0 \quad \text{and} \quad -c_1 + 2c_2 = 0$$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ -1 & 2 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 5 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 1 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \quad \text{so } c_1 = 0 \text{ and } c_2 = 0.$$

Since the only solution to equation $(*)$ above is the <u>trivial</u> solution, the set $\{v - w, 3v + 2w\}$ is linearly independent.