

Quiz 6

- ① $V = \{at^2 + bt + c \mid a+b-c=2\}$
and $\oplus$ is standard polynomial addition.

V is not closed under $\oplus$.

To prove this, it is sufficient to provide a counter-example.

$$\text{let } u = 2t^2 \text{ and } v = 2t$$

then $u, v \in V$ but

$$u \oplus v = 2t^2 + 2t \notin V \text{ since } 2+2-0=4 \neq 2.$$

Hence V is not closed under $\oplus$.

- ② $W = \{at^2 + bt + c \mid a+b-c=0\}$
and $\oplus$ is standard polynomial addition.

W is closed under $\oplus$. To prove this

we must show that $u+v \in W$

for all $u, v \in W$. So, we must be general.

$$\text{let } u = a_1t^2 + b_1t + c_1 \in W \text{ and } (a_1+b_1-c_1=0) \\ v = a_2t^2 + b_2t + c_2 \in W \quad (a_2+b_2-c_2=0)$$

$$\text{Now } u+v = (a_1+a_2)t^2 + (b_1+b_2)t + (c_1+c_2) \in W$$

$$\text{since } (a_1+a_2) + (b_1+b_2) - (c_1+c_2)$$

$$= (a_1+b_1-c_1) + (a_2+b_2-c_2) = 0+0=0$$

Hence W
is closed
under $\oplus$