Math 344 Quiz 3 9/4/15

Find A if $A^{-1} = \begin{bmatrix} 1 & 3 \\ -2 & -7 \end{bmatrix}$

since $(A^{-1})^{-1} = A$, we proceed as in finding

A^{-1} given A.

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ -2 & -7 & 0 & 1 \end{array}\right] \xrightarrow{R_2+2R_1} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{array}\right]$$

$$\xrightarrow{-1\cdot R_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & -1 \end{array}\right] \xrightarrow{R_1-3R_2} \left[\begin{array}{cc|cc} 1 & 0 & 7 & 3 \\ 0 & 1 & -2 & -1 \end{array}\right]$$

Thus $A = \begin{bmatrix} 7 & 3 \\ -2 & -1 \end{bmatrix}$

General formula:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $\det(A) = ad-bc \neq 0$

then $A^{-1} = \dfrac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

In our case, $\det(A^{-1}) = (1)(-7) - (3)(-2) = -7+6 = -1$

and $A = \dfrac{1}{\det(A^{-1})} \begin{bmatrix} -7 & -3 \\ 2 & 1 \end{bmatrix} = -1 \begin{bmatrix} -7 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ -2 & -1 \end{bmatrix}$.