Linear Algebra HW #12 Key

1. $A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$ $\quad (\lambda I_2 - A) = \begin{bmatrix} \lambda-1 & -4 \\ -1 & \lambda+2 \end{bmatrix}$

$\det[\lambda I_2 - A] = (\lambda+2)(\lambda-1) - 4 = (\lambda+3)(\lambda-2) = 0$

$\lambda = -3 \quad \lambda = 2$

$\dim(E_2) = 1$

$\dim(E_3) = 1$

$\Rightarrow$ A is diagonalizable

T2. If A is diagonalizable, then there is a nonsingular matrix P so that $P^{-1}AP = D$, a diagonal matrix. Then, $A^{-1} = PD^{-1}P^{-1} = (P^{-1})^{-1}D^{-1}P^{-1}$. Since $D^{-1}$ is a diagonal matrix, we conclude that $A^{-1}$ is diagonalizable.

2a. $A^{-1} = A^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

6. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\quad \lambda I_3 - A = \begin{bmatrix} \lambda & 0 & -1 \\ 0 & \lambda & 0 \\ -1 & 0 & \lambda \end{bmatrix}$

$$E_{-1} = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \vec{X} = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} = t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$E_0 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \vec{X} = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix} = r\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \vec{X} = \begin{bmatrix} s \\ 0 \\ s \end{bmatrix} = s\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$\boxed{T7.}$ $(A^{-1})^T A^{-1} = (A^T)^{-1} A^{-1} = (A^T A)^{-1} = (I_n)^{-1} = I_n$