Linear Algebra Homework #11 Answer key

3. Find the characteristic polynomial

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -1 & 3 & 2 \end{bmatrix} \qquad \lambda I - A = \begin{bmatrix} \lambda-1 & -2 & -1 \\ 0 & \lambda-1 & -2 \\ 1 & -3 & \lambda-2 \end{bmatrix}$$

$$\det(\lambda I - A) = \lambda-1 \det\begin{bmatrix} \lambda-1 & -2 \\ -3 & \lambda-2 \end{bmatrix} + \det\begin{bmatrix} -2 & -1 \\ \lambda-1 & -2 \end{bmatrix}$$

$$= \lambda-1\left((\lambda-2)(\lambda-1)-6\right) + 4 + \lambda - 1$$

$$= \boxed{\lambda^3 - 4\lambda^2 + 7}$$

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4. $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \qquad \lambda I - A = \begin{bmatrix} \lambda-2 & -1 \\ 1 & \lambda-3 \end{bmatrix}$

$$\det(\lambda I - A) = (\lambda-2)(\lambda-3) + 1$$

$$= \boxed{\lambda^2 - 5\lambda + 7}$$

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1. $\lambda I - A = \begin{bmatrix} \lambda-1 & 0 & 0 \\ 1 & \lambda-3 & 0 \\ -3 & -2 & \lambda+2 \end{bmatrix}$

$$\det(\lambda I - A) = \lambda-1 \det\begin{bmatrix} \lambda-3 & 0 \\ -2 & \lambda+2 \end{bmatrix}$$

$$= \lambda-1(\lambda-3)(\lambda+2) = 0$$

$$\Rightarrow \boxed{\lambda = 1,\ \lambda = 3,\ \lambda = -2}$$

Eigenvectors:

$\lambda = 1 \Rightarrow t\begin{bmatrix} 6 \\ 3 \\ 8 \end{bmatrix} \qquad \lambda = 3 \Rightarrow t\begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$

$\lambda = -2 \Rightarrow t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\boxed{T1.}$ Let u and v be in S so that $Au = \lambda_j u$ and $Av = \lambda_j v$. Then, $A(u+v) = \lambda_j u + \lambda_j v = \lambda_j (u+v)$. So, $(u+v) \in S$. Moreover, if $c \in \mathbb{R}$, then $A c(u) = cA(u) = c(\lambda_j u) = \lambda_j (cu)$. So, $cu \in S$.

$\boxed{T5.}$ $A^k \vec{x} = A^{k-1}(A\vec{x}) = A^{k-1}(\lambda \vec{x}) = \lambda A^{k-1}(\vec{x}) = \dots = \lambda^k \vec{x}$.
