Math 344 Homework #9 Answer key

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a. (a,b,c,d) where $d=a+b$

$= \{(a,0,0,a) + (0,b,0,b) +, (0,0,c,0)\}$

$= \{a(1,0,0,1) + b(0,1,0,1) + c(0,0,1,0)\}$

$\dim(V) = 3.$

b. (a,b,c,d) where $c=a-b$ and $d=a+b$

$= \{(a,b,a-b,a+b)\}$

$= \{a(1,0,1,1) + b(0,1,-1,1)\}$

$\dim(V) = 2$

T12

If A is nonsingular, then the linear system $A\vec{x}=0$ has only the trivial solution $\vec{x}=0$.

Let $c_1Av_1 + c_2Av_2 + \dots + c_nAv_n = 0$.

Then, $A[c_1v_1 + c_2v_2 + \dots + c_nv_n] = 0$.

Thus, it must be that $c_1v_1 + c_2v_2 + \dots + c_nv_n = 0$.

since $\{v_1, v_2, \dots, v_n\}$ is linearly independent, it follows that $c_1 = c_2 = \dots = c_n$. Hence, $\{Av_1, \dots Av_n\}$ is linearly independent as desired.

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$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 2 & 1 & 3 & 0 \\ 3 & 1 & 2 & 0 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right]$$

$$\Rightarrow R_3 - R_2 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$X_1 = 0$
$X_2 = 0$ $\Rightarrow$ only solution is the trivial solution.
$X_3 = 0$

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$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 2 & 3 & 2 & 0 & 0 \\ 3 & 4 & 1 & 1 & 0 \\ 1 & 1 & -1 & 1 & 0 \end{array}\right] \xrightarrow[R_4-R_1]{\substack{R_2-2R_1 \\ R_3-3R_1}} \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 0 & -1 & -4 & 2 & 0 \\ 0 & -2 & -8 & 4 & 0 \\ 0 & -1 & -4 & 2 & 0 \end{array}\right]$$

$$\xrightarrow[\substack{-\frac{1}{2}R_3 \\ -1R_4}]{-1R_2} \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 0 & 1 & 4 & -2 & 0 \\ 0 & 1 & 4 & -2 & 0 \\ 0 & 1 & 4 & -2 & 0 \end{array}\right] \xrightarrow[R_4-R_2]{R_3-R_2} \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 0 & 1 & 4 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$X_3 = t$ $\quad X_2 = 2s - 4t$
$X_4 = s$ $\quad X_1 = 5t - 3s$

$$\begin{bmatrix} 5t-3s \\ 2s-4t \\ t \\ s \end{bmatrix} = \begin{bmatrix} 5t \\ -4t \\ t \\ 0 \end{bmatrix} + \begin{bmatrix} -3s \\ 2s \\ 0 \\ s \end{bmatrix}$$

$$= t \begin{bmatrix} 5 \\ -4 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \quad \text{Basis: } \left\{ \begin{bmatrix} 5 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

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$$A = \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 2 & 1 & -4 & -5 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 0 & -3 & -6 & -11 & 0 \\ 0 & -1 & -1 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_3 \Leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 0 & -1 & -1 & -3 & 0 \\ 0 & -3 & -6 & -11 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[-1(R_3)]{-1(R_2)} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 3 & 6 & 11 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_3 - 3R_2} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{3}R_3} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 2/3 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_4 - R_3} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 1/3 & 0 \end{array}\right]$$

$$\xrightarrow{3R_4} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Nullity = 0

Rank = 4

Nullity + Rank = 4 + 0 = 4.