Linear Algebra HW #7 Answer key

$\boxed{5b.}$ $u = (a, b, c)$, where $c = a+b$

α). $(a, b, c) \oplus (a', b', c') = (a+a', b+b', c+c')$

$= (a+a', b+b', (a+b)+(a'+b'))$ ✓

β). $c \odot (a, b, a+b) = (ca, cb, c(a+b))$ ✓

$\Rightarrow$ Thus, this is a subspace of $\mathbb{R}^3$.

(8a.) $u = (a, b, c, d)$, where $a = b = 0$.

α). $(0, 0, c, d) \oplus (0, 0, c', d') = (0, 0, c+c', d+d')$ ✓

β). let m be a scalar

$m \odot (0, 0, c, d) = (0, 0, mc, md)$ ✓

$\Rightarrow$ Thus, this is a subspace of $\mathbb{R}^4$

(9a.) $a_2t^2 + a_1t + a_0$, where $a_0 = 0$

α). $(a_2t^2 + a_1t + 0) \oplus (b_2t^2 + b_1t + 0)$

$= (a_2 + b_2)t^2 + (a_1 + b_1)t$ ✓

β). $c \odot (a_2t^2 + a_1t + 0) = ca_2t^2 + ca_1t$ ✓

$\Rightarrow$ Thus, this is a subspace of P_2.

(9c.) $a_2t^2 + a_1t + a_0$, where $a_2 + a_1 = a_0$

α). $(a_2t^2 + a_1t + a_2 + a_1) \oplus (b_2t^2 + b_1t + b_2 + b_1)$

$= (a_2 + b_2)t^2 + (a_1 + b_1)t + (a_2 + a_1 + b_2 + b_1)$ ✓

β). $c \odot (a_2t^2 + a_1t + a_2 + a_1)$

$= ca_2t^2 + ca_1t + ca_2 + ca_1$

$\boxed{27a.}$ $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ -1 & -2 & 0 & -3 \\ 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$

$\rightarrow R_2+R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right] \Rightarrow$ no solution

$\Rightarrow$ Thus, $p(t)$ does not belong to the span $\{p_1(t), p_2(t), p_3(t)\}$

$\boxed{27b.}$ $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ -1 & -2 & 0 & -1 \\ 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$

$\rightarrow R_2+R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right] \Rightarrow$ no solution

$\Rightarrow$ Thus, $p(t)$ does not belong to the span $\{p_1(t), p_2(t), p_3(t)\}$