Linear Algebra HW #6 Answer key

(1.) α) $(x,y) \oplus (x',y') = (x+x', y+y') \in V$. since positive + positive = positive

$\Rightarrow$ closed under $\oplus$.

β) $c \odot (x,y) = (cx, cy)$

$\Rightarrow$ let $c = -1$, then $-1 \odot (x,y) = (-x,-y) \notin V$.

$\Rightarrow$ Not closed under $\odot$.

(2.) α) $(0,y,z) \oplus (0,y',z') = (0, y+y', z+z') \in V$

$\Rightarrow$ closed under $\oplus$

β) $c \odot (0,y,z) = (0, 0, cz) \in V$

$\Rightarrow$ closed under $\odot$

(12.) $\Rightarrow$ Closed under $\oplus$

$\Rightarrow$ Fails ~~to be closed under $\odot$ with~~ the following properties:

(e) $c \odot ((x,y,z) \oplus (x',y',z')) = c \odot (x+x', y+y', z+z')$

$= (x+x', 1, z+z')$

$c \odot (x,y,z) \oplus c \odot (x',y',z') = (x, 1, z) \oplus (x', 1, z')$

$= (x+x', 2, z+z')$

$(x+x', 2, z+z') \neq (x+x', 1, z+z') \Rightarrow$ property e fails

(f.) $(c+d) \odot (x,y,z) = (x, 1, z)$

$c \odot (x,y,z) \oplus d \odot (x,y,z) = (x,1,z) + (x,1,z)$

$= (2x, 2, 2z)$

$(x, 1, z) \neq (2x, 2, 2z)$
$\Rightarrow$ property f fails

h. $1 \odot (x, y, z) = (x, 1, z) \neq (x, y, z)$ if $y \neq 1$

$\Rightarrow$ property h fails

---

16. $\Rightarrow$ Closed under $+$

$\Rightarrow$ ~~NOT closed under $\odot$ with~~ The following property fails:

h. $1 \odot (x, y) = (0, 0) \neq (x, y)$ if either $x \neq 0$ or $y \neq 0$

$\Rightarrow$ Thus, property h fails

---

20. a. There are infinitely many vectors in V. b/c if $v \neq \vec{0}$ then $cv \in V$ for all $c \in \mathbb{R}$.

b. The only vector space having a finite number of vectors is $\{0\}$.