Math 344 Homework #5 Answer key

pg. 244

(1.) Find the length of the following vectors:

(a.) $(2,3,4)$

$\|\vec{u}\| = \sqrt{2^2+3^2+4^2} = \sqrt{4+9+16} = \boxed{\sqrt{29}}$

(b.) $(0,-1,2,3)$

$\|\vec{u}\| = \sqrt{0+(-1)^2+2^2+3^2} = \sqrt{1+4+9} = \boxed{\sqrt{14}}$

(c.) $(-1,-2,0)$

$\|\vec{u}\| = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \boxed{\sqrt{5}}$

(d.) $(1,2,-3,-4)$

$\|\vec{u}\| = \sqrt{(1)^2+(2)^2+(-3)^2+(-4)^2} = \sqrt{1+4+9+16} = \boxed{\sqrt{30}}$

(5.) If possible, find scalars c_1, c_2, c_3, not all zero so that:

$$c_1\begin{bmatrix}1\\2\\-1\end{bmatrix} + c_2\begin{bmatrix}1\\3\\-2\end{bmatrix} + c_3\begin{bmatrix}3\\7\\-4\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&1&3&0\\2&3&7&0\\-1&-2&-4&0\end{array}\right] \xRightarrow[-1(R_3)]{R_2-2R_1} \left[\begin{array}{ccc|c}1&1&3&0\\0&1&1&0\\1&2&4&0\end{array}\right]$$

$$\xRightarrow{R_3-R_1} \left[\begin{array}{ccc|c}1&1&3&0\\0&1&1&0\\0&1&1&0\end{array}\right] \xRightarrow{R_3-R_2} \left[\begin{array}{ccc|c}1&1&3&0\\0&1&1&0\\0&0&0&0\end{array}\right]$$

Let $c_3 = t$.

Then $c_1 = -2t$ and $c_2 = -t$.

17. Find all constants a such that $\vec{u}\cdot\vec{v}=0$, where $\vec{u}=(a,2,1,a)$ and $\vec{v}=(a,-1,-2,-3)$.

$$\vec{u}\cdot\vec{v}=0$$

$$(a,2,1,a)\cdot(a,-1,-2,-3)=a^2-2-2-3a=0$$

$$a^2-3a-4=0$$

$$(a-4)(a+1)=0$$

$$\boxed{a=4 \text{ or } a=-1}$$

21. Find the cosine of the angle between each pair of vectors $\vec{u}$ and $\vec{v}$.

a.) $\vec{u}=(2,3,1)$ $\vec{v}=(3,-2,0)$

$$\cos\theta=\frac{u\cdot v}{\|u\|\cdot\|v\|}=\frac{6-6+0}{\sqrt{14}(\sqrt{13})}=\frac{0}{\sqrt{182}}=0$$

$$\boxed{\cos\theta=0}$$

b.) $\vec{u}=(1,2,-1,3)$ $\vec{v}=(0,0,-1,-2)$

$$\cos\theta=\frac{0+0+1-6}{\sqrt{15}\ \sqrt{5}}=\frac{-5}{\sqrt{75}}$$

$$\cos\theta=\frac{-5}{\sqrt{75}}=\boxed{\cos\theta=\frac{-1}{\sqrt{3}}}$$

c.) $\vec{u}=(2,0,1)$ $\vec{v}=(2,2,-1)$

$$\cos\theta=\frac{4+0-1}{(\sqrt{5})(3)}=\frac{\cancel{3}}{\cancel{3}\sqrt{5}}=\frac{1}{\sqrt{5}}$$

$\cos\theta = 1$

(d.) $u=(0,4,2,3)$ $v=(0,-1,2,0)$

$\cos\theta = \frac{0-4+4+0}{\sqrt{29}(\sqrt{5})} = 0$

$\boxed{\cos\theta = 0}$

(24.) Find c so that the vector $v=(2,c,3)$ is orthogonal to $w=(1,-2,1)$.

$\vec{v}\cdot\vec{w}=0$

$(2,c,3)\cdot(1,-2,1)=0$

$2+-2c+3=0$

$-2c+5=0$

$-2c=-5$

$\boxed{c=\frac{5}{2}}$