

Math 344 Homework #3 key

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3a. Compute the determinant

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 2 \\ 3 & 2 & 0 \end{bmatrix}$$

$$\det(A) = 3 \det \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix} - 2 \det \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} + 0 \det \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}$$

$$\det(A) = 3[-2 - 15] - 2[2 - 6]$$

$$\det(A) = -51 + 8 = \boxed{-43}$$

3c. Compute the determinant

$$A = \begin{bmatrix} 4 & -2 & 0 \\ 0 & 2 & 4 \\ -1 & -1 & -3 \end{bmatrix}$$

$$\det(A) = 4 \det \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix} - 0 \det \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix} + (-1) \det \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$$

$$\det(A) = 4(-6 + 4) - 1(-8 - 0)$$

$$\det(A) = -8 + 8 = \boxed{0}$$

(17a.) Find all values of λ for which

$$\det \begin{bmatrix} \lambda-1 & -4 \\ 0 & \lambda-4 \end{bmatrix} = 0.$$

$$\det(A) = \lambda-1(\lambda-4) - 0 = 0$$
$$\lambda-1=0 \quad \lambda-4=0$$
$$\boxed{\lambda=1 \text{ and } \lambda=4}$$

(17b.) Find all values of λ for which $\det(\lambda I_3 - A) = 0$

$$\lambda I_3 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \quad A = \begin{bmatrix} -3 & -1 & -3 \\ 0 & 3 & 0 \\ -2 & -1 & -2 \end{bmatrix}$$

$$\lambda I_3 - A = \begin{bmatrix} \lambda+3 & 1 & 3 \\ 0 & \lambda-3 & 0 \\ -2 & 1 & \lambda+2 \end{bmatrix}$$

$$\det(\lambda I_3 - A) = \lambda-3((\lambda+3)(\lambda+2)-6) = 0$$
$$= \lambda-3(\lambda^2+5\lambda+6-6) = 0$$
$$= (\lambda-3)(\lambda^2+5\lambda) = 0$$
$$= (\lambda-3)\lambda(\lambda+5) = 0$$

$$\boxed{\lambda=3, \lambda=0, \lambda=-5}$$

(T10.) If $\det(A) \neq 0$, then A is nonsingular and A^{-1} exists.

$$\text{So } B = A^{-1}AB = A^{-1}AC = C.$$

$$\text{Thus, } B = C.$$