

Math 344 Homework #2 Key

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#11) Show that $AB = AC$

$$AB = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2+6 & -6+0 \\ -2-6 & 6+0 \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ -8 & 6 \end{bmatrix}$$

$$AC = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -4 & -3 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 8+0 & 6-12 \\ -8+0 & -6+12 \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ -8 & 6 \end{bmatrix}$$

$$\text{Since } \begin{bmatrix} 8 & -6 \\ -8 & 6 \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ -8 & 6 \end{bmatrix}, \quad AB = AC$$

#12) show that if $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A^2 = I_2$.

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Find the inverses of the given matrices if possible.

$$(5b.) \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \xRightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\Rightarrow -1(R_2) \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \xRightarrow{\substack{R_1 - 2R_2 \\ R_3 - R_2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right]$$

$$\Rightarrow \substack{R_1 - R_3 \\ R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right] \quad A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

(6b.) $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array} \right] \xRightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$

$\xRightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$

$\xRightarrow{R_2 - 3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & 3 & 1 & -3 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right] \xRightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 3/2 & 1/2 & -3/2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$

$A^{-1} = \left[\begin{array}{ccc} 1 & -1 & 0 \\ 3/2 & 1/2 & -3/2 \\ -1 & 0 & 1 \end{array} \right]$

(T.1.) suppose that A and B are square matrices and $AB=O$. If B is nonsingular, find A.

B is nonsingular, so, B^{-1} exists and

$$A = A I_n = A (B B^{-1}) = (AB) B^{-1} = O (B^{-1}) = O.$$

SO $A = O$ vector.