Math 344 HW #1 key

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#22. (A.) zero points

(B.) Infinitely many points

(C.) zero points

#23.

$$\begin{aligned} 5x + 4y &= 180 \\ 4x + 2y &= 120 \end{aligned} \Longrightarrow \begin{aligned} 5x + 4y &= 180 \\ -(8x + 4y &= 240) \\ \hline -3x &= -60 \\ x &= 20 \text{ tons} \end{aligned}$$

$$5(20) + 4y = 180$$
$$100 + 4y = 180$$
$$4y = 80$$
$$y = 20 \text{ tons}$$

$$\boxed{\text{Solution: } x = 20 \text{ tons},\ y = 20 \text{ tons}}$$

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#2. If $\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$ find a, b, c, d.

$$\begin{aligned} a + b &= 4 \\ +\ a - b &= 2 \\ \hline 2a &= 6 \\ a &= 3 \end{aligned}$$

$$3 + b = 4$$
$$b = 1$$

$$\begin{aligned} c + d &= 6 \\ +\ c - d &= 10 \\ \hline 2c &= 16 \\ c &= 8 \end{aligned}$$

$$8 + d = 6$$
$$d = -2$$

$$\boxed{\text{Solutions: } a = 3,\ b = 1,\ c = 8,\ d = -2}$$

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$\boxed{\#11.}$ Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

Show that $AB \neq BA$.

$$\Rightarrow AB = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2-6 & -1+8 \\ 6-6 & -3+8 \end{bmatrix} = \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2-3 & 4-2 \\ -3+12 & -6+8 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix}$$

Since $\begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix}$, $AB \neq BA$ as desired.

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$\boxed{\#21.}$ (A)

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 1 & -2 & 1 & 1 & 8 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-R_1} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -2 & -5 & -10 & -5 \\ 0 & -3 & -1 & -2 & -38 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & 5/2 & 5 & 19 \\ 0 & -3 & -1 & -2 & -5 \end{array}\right]$$

$$\xrightarrow{R_3+3R_2} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & 5/2 & 5 & 19 \\ 0 & 0 & 13/2 & 13 & 52 \end{array}\right] \xrightarrow{\frac{2}{13}R_3} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & 5/2 & 5 & 19 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right]$$

$z + 2w = 8$ let $\boxed{w = t}$

$\boxed{z = 8 - 2t}$

$y + \frac{5}{2}(8-2t) + 5t = 19$

$y + 20 - \cancel{5t} + \cancel{5t} = 19$

$\boxed{y = -1}$

$x - 1 + 2(8-2t) + 3t = 13$

$x - 1 + 16 - 4t + 3t = 13$

$x - t = -2$

$\boxed{x = t - 2}$

Solutions: $(x = t-2), (y = -1), (z = 8-2t), (w = t)$

(B) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 1 & 1 & 2 \end{array}\right] \xRightarrow[R_3-2R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & -1 & -1 & 0 \end{array}\right]$

$R_2 \rightleftarrows R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -3 & 2 \end{array}\right] \Rightarrow -1R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & -2 \end{array}\right]$

$\xRightarrow{\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2/3 \end{array}\right]$

$z = -\frac{2}{3}$

$y - \frac{2}{3} = 0$

$y = \frac{2}{3}$

$x - \frac{2}{3} + \frac{2}{3} = 1$

$x = 1$

$\boxed{\text{Solutions: } x=1,\ y=\frac{2}{3},\ z=-\frac{2}{3}}$

(C) $\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 1 & 1 & -1 & -1 & -1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right] \xRightarrow{R_3 \rightleftarrows R_1} \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 3 & -2 & 1 & -6 & -2 \\ 2 & 1 & 1 & -2 & 1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right]$

$R_2 - 3R_1$, $R_3 - 2R_1$, $R_4 - 6R_1$, $R_5 - 5R_1$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & -6 & 7 & -3 & 8 \end{array}\right] \xRightarrow{R_4 - R_5} \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & -6 & 7 & -3 & 8 \end{array}\right]$

$0 \neq -4 \Rightarrow$ $\boxed{\text{Therefore, there is no solution!}}$

(D) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 2 & 1 & -1 & 1 & 3 \\ 1 & -1 & 0 & 1 & -2 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 0 & -3 & -7 & 3 & 3 \\ 0 & -3 & -3 & 2 & -2 \end{array}\right] \ldots$

$\xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 0 & 1 & 7/3 & -1 & -1 \\ 0 & -3 & -3 & 2 & -2 \end{array}\right] \xrightarrow{R_3+3R_2} \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 0 & 1 & 7/3 & -1 & -1 \\ 0 & 0 & 4 & -1 & -5 \end{array}\right]$

$\xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 0 \\ 0 & 1 & 7/3 & -1 & -1 \\ 0 & 0 & 1 & -1/4 & -5/4 \end{array}\right]$

$z - \frac{1}{4}w = -\frac{5}{4}$

$z - \frac{1}{4}t = -\frac{5}{4}$

$z = \frac{1}{4}t - \frac{5}{4}$

let $w = t$

$y + \frac{7}{3}\left(\frac{1}{4}t - \frac{5}{4}\right) - t = -1$

$y + \frac{7}{12}t - \frac{35}{12} - t = -1$

$y - \frac{5}{12}t = \frac{23}{12}$

$y = \frac{5}{12}t + \frac{23}{12}$

$x + 2\left(\frac{5}{12}t + \frac{23}{12}\right) + 3\left(\frac{1}{4}t - \frac{5}{4}\right) - t = 0$

$x + \frac{5}{6}t + \frac{23}{6} + \frac{3}{4}t - \frac{15}{4} - t = 0$

$x + \frac{7}{12}t + \frac{1}{12} = 0$

$x = -\frac{7}{12}t - \frac{1}{12}$

Solutions: $x = -\frac{7}{12}t - \frac{1}{12}$

$y = \frac{5}{12}t + \frac{23}{12}$

$z = \frac{1}{4}t - \frac{5}{4}$

$w = t$