

since we must have $x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$. When $x_3 = 10$, we have

$$x_1 = 5, \quad x_2 = 10, \quad x_3 = 10$$

while

$$x_1 = \frac{13}{2}, \quad x_2 = 13, \quad x_3 = 7$$

when $x_3 = 7$. The reader should observe that one solution is just as good as the other. There is no best solution unless additional information or restrictions are given. ■

Key Terms

Linear equation

Unknowns

Solution to a linear equation

Linear system

Solution to a linear system

Method of elimination

Unique solution

No solution

Infinitely many solutions

Manipulations on a linear system

1.1 Exercises

In Exercises 1 through 14, solve the given linear system by the method of elimination.

1.
$$\begin{aligned} x + 2y &= 8 \\ 3x - 4y &= 4 \end{aligned}$$

2.
$$\begin{aligned} 2x - 3y + 4z &= -12 \\ x - 2y + z &= -5 \\ 3x + y + 2z &= 1 \end{aligned}$$

3.
$$\begin{aligned} 3x + 2y + z &= 2 \\ 4x + 2y + 2z &= 8 \\ x - y + z &= 4 \end{aligned}$$

4.
$$\begin{aligned} x + y &= 5 \\ 3x + 3y &= 10 \end{aligned}$$

5.
$$\begin{aligned} 2x + 4y + 6z &= -12 \\ 2x - 3y - 4z &= 15 \\ 3x + 4y + 5z &= -8 \end{aligned}$$

6.
$$\begin{aligned} x + y - 2z &= 5 \\ 2x + 3y + 4z &= 2 \end{aligned}$$

7.
$$\begin{aligned} x + 4y - z &= 12 \\ 3x + 8y - 2z &= 4 \end{aligned}$$

8.
$$\begin{aligned} 3x + 4y - z &= 8 \\ 6x + 8y - 2z &= 3 \end{aligned}$$

9.
$$\begin{aligned} x + y + 3z &= 12 \\ 2x + 2y + 6z &= 6 \end{aligned}$$

10.
$$\begin{aligned} x + y &= 1 \\ 2x - y &= 5 \\ 3x + 4y &= 2 \end{aligned}$$

11.
$$\begin{aligned} 2x + 3y &= 13 \\ x - 2y &= 3 \\ 5x + 2y &= 27 \end{aligned}$$

12.
$$\begin{aligned} x - 5y &= 6 \\ 3x + 2y &= 1 \\ 5x + 2y &= 1 \end{aligned}$$

13.
$$\begin{aligned} x + 3y &= -4 \\ 2x + 5y &= -8 \\ x + 3y &= -5 \end{aligned}$$

14.
$$\begin{aligned} 2x + 3y - z &= 6 \\ 2x - y + 2z &= -8 \\ 3x - y + z &= -7 \end{aligned}$$

15. Given the linear system

$$\begin{aligned} 2x - y &= 5 \\ 4x - 2y &= t, \end{aligned}$$

- (a) determine a value of t so that the system has a solution.
- (b) determine a value of t so that the system has no solution.

(c) how many different values of t can be selected in part (b)?

16. Given the linear system

$$\begin{aligned} 2x + 3y - z &= 0 \\ x - 4y + 5z &= 0, \end{aligned}$$

- (a) verify that $x_1 = 1, y_1 = -1, z_1 = -1$ is a solution.
- (b) verify that $x_2 = -2, y_2 = 2, z_2 = 2$ is a solution.
- (c) is $x = x_1 + x_2 = -1, y = y_1 + y_2 = 1$, and $z = z_1 + z_2 = 1$ a solution to the linear system?
- (d) is $3x, 3y, 3z$, where x, y , and z are as in part (c), a solution to the linear system?

17. Without using the method of elimination, solve the linear system

$$\begin{aligned} 2x + y - 2z &= -5 \\ 3y + z &= 7 \\ z &= 4. \end{aligned}$$

18. Without using the method of elimination, solve the linear system

$$\begin{aligned} 4x &= 8 \\ -2x + 3y &= -1 \\ 3x + 5y - 2z &= 11. \end{aligned}$$

19. Is there a value of r so that $x = 1, y = 2, z = r$ is a solution to the following linear system? If there is, find it.

$$\begin{aligned} 2x + 3y - z &= 11 \\ x - y + 2z &= -7 \\ 4x + y - 2z &= 12 \end{aligned}$$

20. Is there a value of r so that $x = r$, $y = 2$, $z = 1$ is a solution to the following linear system? If there is, find it.

$$\begin{aligned} 3x - 2z &= 4 \\ x - 4y + z &= -5 \\ -2x + 3y + 2z &= 9 \end{aligned}$$

21. Describe the number of points that simultaneously lie in each of the three planes shown in each part of Figure 1.2.
22. Describe the number of points that simultaneously lie in each of the three planes shown in each part of Figure 1.3.

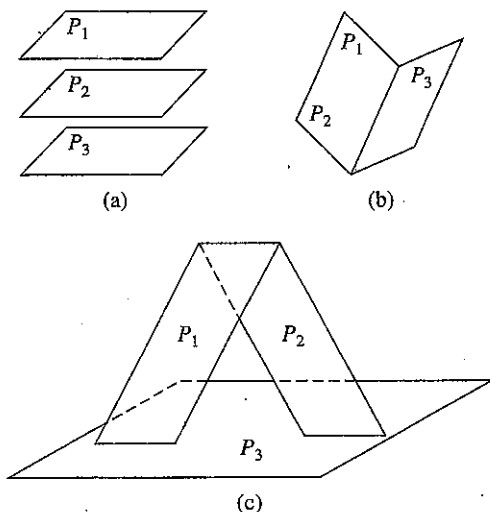


Figure 1.3

23. An oil refinery produces low-sulfur and high-sulfur fuel. Each ton of low-sulfur fuel requires 5 minutes in the blending plant and 4 minutes in the refining plant; each ton of high-sulfur fuel requires 4 minutes in the blending plant and 2 minutes in the refining plant. If the blending plant is available for 3 hours and the refining plant is available for 2 hours, how many tons of each type of fuel should be manufactured so that the plants are fully utilized?

24. A plastics manufacturer makes two types of plastic: regular and special. Each ton of regular plastic requires 2 hours in plant A and 5 hours in plant B; each ton of special plastic requires 2 hours in plant A and 3 hours in plant B. If plant A is available 8 hours per day and plant B is available 15 hours per day, how many tons of each type of plastic can be made daily so that the plants are fully utilized?

25. A dietician is preparing a meal consisting of foods A, B, and C. Each ounce of food A contains 2 units of protein, 3 units of fat, and 4 units of carbohydrate. Each ounce of food B contains 3 units of protein, 2 units of fat, and 1 unit of carbohydrate. Each ounce of food C contains 3 units of protein, 3 units of fat, and 2 units of carbohydrate. If the meal must provide exactly 25 units of protein, 24 units of fat, and 21 units of carbohydrate, how many ounces of each type of food should be used?

26. A manufacturer makes 2-minute, 6-minute, and 9-minute film developers. Each ton of 2-minute developer requires 6 minutes in plant A and 24 minutes in plant B. Each ton of 6-minute developer requires 12 minutes in plant A and 12 minutes in plant B. Each ton of 9-minute developer requires 12 minutes in plant A and 12 minutes in plant B. If plant A is available 10 hours per day and plant B is available 16 hours per day, how many tons of each type of developer can be produced so that the plants are fully utilized?

27. Suppose that the three points $(1, -5)$, $(-1, 1)$, and $(2, 7)$ lie on the parabola $p(x) = ax^2 + bx + c$.

- (a) Determine a linear system of three equations in three unknowns that must be solved to find a , b , and c .
- (b) Solve the linear system obtained in part (a) for a , b , and c .

28. An inheritance of \$24,000 is to be divided among three trusts, with the second trust receiving twice as much as the first trust. The three trusts pay interest at the rates of 9%, 10%, and 6% annually, respectively, and return a total in interest of \$2210 at the end of the first year. How much was invested in each trust?

Theoretical Exercises

- T.1. Show that the linear system obtained by interchanging two equations in (2) has exactly the same solutions as (2).
- T.2. Show that the linear system obtained by replacing an equation in (2) by a nonzero constant multiple of the equation has exactly the same solutions as (2).
- T.3. Show that the linear system obtained by replacing an

equation in (2) by itself plus a multiple of another equation in (2) has exactly the same solutions as (2).

- T.4. Does the linear system

$$\begin{aligned} ax + by &= 0 \\ cx + dy &= 0 \end{aligned}$$

always have a solution for any values of a , b , c , and d ?

Key Terms

Matrix
 Rows
 Columns
 Size of a matrix
 Square matrix
 Main diagonal of a matrix
 Element (or entry) of a matrix
 i / j th element
 (i, j) entry

n -vector (or vector)
 Diagonal matrix
 Scalar matrix
 $\mathbf{0}$, the zero vector
 R^n , the set of all n -vectors
 Google®
 Equal matrices
 Matrix addition
 Scalar multiplication

Scalar multiple of a matrix
 Difference of matrices
 Linear combination of matrices
 Transpose of a matrix
 Bit
 Bit (or Boolean) matrix
 Upper triangular matrix
 Lower triangular matrix

1.2 Exercises

1. Let

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 7 & 3 & 2 \\ -4 & 3 & 5 \\ 6 & 1 & -1 \end{bmatrix}$$

 (a) What is a_{12} , a_{22} , a_{23} ?

 (b) What is b_{11} , b_{31} ?

 (c) What is c_{13} , c_{31} , c_{33} ?

2. If

$$\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$$

 find a , b , c , and d .

3. If

$$\begin{bmatrix} a+2b & 2a-b \\ 2c+d & c-2d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -3 \end{bmatrix}$$

 find a , b , c , and d .

In Exercises 4 through 7, let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$$

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$$

$$\text{and } O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4. If possible, compute the indicated linear combination:

 (a) $C + E$ and $E + C$

 (b) $A + B$

 (c) $D - F$

 (d) $-3C + 5O$

 (e) $2C - 3E$

 (f) $2B + F$

5. If possible, compute the indicated linear combination:

 (a) $3D + 2F$

 (b) $3(2A)$ and $6A$

 (c) $3A + 2A$ and $5A$

 (d) $2(D + F)$ and $2D + 2F$

 (e) $(2 + 3)D$ and $2D + 3D$

 (f) $3(B + D)$

6. If possible, compute:

 (a) A^T and $(A^T)^T$

 (b) $(C + E)^T$ and $C^T + E^T$

 (c) $(2D + 3F)^T$

 (d) $D - D^T$

 (e) $2A^T + B$

 (f) $(3D - 2F)^T$

7. If possible, compute:

 (a) $(2A)^T$

 (b) $(A - B)^T$

 (c) $(3B^T - 2A)^T$

 (d) $(3A^T - 5B^T)^T$

 (e) $(-A)^T$ and $-(A^T)$

 (f) $(C + E + F)^T$

 8. Is the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ a linear combination of the

 matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? Justify your answer.

 9. Is the matrix $\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix}$ a linear combination of the

 matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? Justify your answer.

10. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & -2 & 3 \\ 5 & 2 & 4 \end{bmatrix} \quad \text{and} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

 If λ is a real number, compute $\lambda I_3 - A$.

Exercises 11 through 15 involve bit matrices.

11. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, and

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$
. Compute each of the following.

- (a)
- $A + B$
- (b)
- $B + C$
- (c)
- $A + B + C$
-
- (d)
- $A + C^T$
- (e)
- $B - C$

12. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, and $D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. Compute each of the following.

- (a)
- $A + B$
- (b)
- $C + D$
- (c)
- $A + B + (C + D)^T$
-
- (d)
- $C - B$
- (e)
- $A - B + C - D$

13. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

- (a) Find
- B
- so that
- $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- .
-
- (b) Find
- C
- so that
- $A + C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
- .

14. Let $u = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$. Find the bit 4-vector v so that $u + v = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$.

15. Let $u = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$. Find the bit 4-vector v so that $u + v = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$.

Theoretical Exercises

T.1. Show that the sum and difference of two diagonal matrices is a diagonal matrix.

T.2. Show that the sum and difference of two scalar matrices is a scalar matrix.

T.3. Let

$$A = \begin{bmatrix} a & b & c \\ c & d & e \\ e & e & f \end{bmatrix}$$

- (a) Compute
- $A - A^T$
- .
-
- (b) Compute
- $A + A^T$
- .
-
- (c) Compute
- $(A + A^T)^T$
- .

T.4. Let O be the $n \times n$ matrix all of whose entries are zero. Show that if k is a real number and A is an $n \times n$ matrix such that $kA = O$, then $k = 0$ or $A = O$.T.5. A matrix $A = [a_{ij}]$ is called **upper triangular** if $a_{ij} = 0$ for $i > j$. It is called **lower triangular** if $a_{ij} = 0$ for $i < j$.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & \cdots & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & \cdots & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & a_{nn} \end{bmatrix}$$

Upper triangular matrix
(The elements below the main diagonal are zero.)

$$\begin{bmatrix} a_{11} & 0 & 0 & \cdots & \cdots & 0 \\ a_{21} & a_{22} & 0 & \cdots & \cdots & 0 \\ a_{31} & a_{32} & a_{33} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ a_{n1} & a_{n2} & a_{n3} & \cdots & \cdots & a_{nn} \end{bmatrix}$$

Lower triangular matrix
(The elements above the main diagonal are zero.)

- (a) Show that the sum and difference of two upper triangular matrices is upper triangular.
-
- (b) Show that the sum and difference of two lower triangular matrices is lower triangular.
-
- (c) Show that if a matrix is both upper and lower triangular, then it is a diagonal matrix.
-
- T.6. (a) Show that if
- A
- is an upper triangular matrix, then
- A^T
- is lower triangular.
-
- (b) Show that if
- A
- is a lower triangular matrix, then
- A^T
- is upper triangular.
-
- T.7. If
- A
- is an
- $n \times n$
- matrix, what are the entries on the main diagonal of
- $A - A^T$
- ? Justify your answer.
-
- T.8. If
- x
- is an
- n
- vector, show that
- $x + 0 = x$
- .

Exercises T.9 through T.18 involve bit matrices.

- T.9. Make a list of all possible bit 2-vectors. How many are there?
-
- T.10. Make a list of all possible bit 3-vectors. How many are there?
-
- T.11. Make a list of all possible bit 4-vectors. How many are there?

EXAMPLE 29

Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ be bit matrices. Then

$$\begin{aligned} AB &= \begin{bmatrix} (1)(0) + (1)(1) & (1)(1) + (1)(1) & (1)(0) + (1)(0) \\ (0)(0) + (1)(1) & (0)(1) + (1)(1) & (0)(0) + (1)(0) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}. \end{aligned}$$

EXAMPLE 30

Let $A = \begin{bmatrix} 1 & 1 & 1 & x \\ 1 & 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ 0 \\ 1 \\ 1 \end{bmatrix}$ be bit matrices. If $AB = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, find x and y .

Solution We have

$$AB = \begin{bmatrix} 1 & 1 & 1 & x \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} y + 1 + x \\ y + 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Then $y + 1 + x = 1$ and $y + 1 = 1$. Using base 2 arithmetic, it follows that $y = 0$ and so then $x = 0$.

Key Terms

Dot product (inner product)
Product of matrices
Coefficient matrix

Augmented matrix
Submatrix
Partitioned matrix

Block multiplication
Summation notation

1.3 Exercises

In Exercises 1 and 2, compute $\mathbf{a} \cdot \mathbf{b}$.

1. (a) $\mathbf{a} = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

(b) $\mathbf{a} = \begin{bmatrix} -3 & -2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $\mathbf{a} = \begin{bmatrix} 4 & 2 & -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$

(d) $\mathbf{a} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

2. (a) $\mathbf{a} = \begin{bmatrix} 2 & -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

(b) $\mathbf{a} = \begin{bmatrix} 1 & -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $\mathbf{a} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

(d) $\mathbf{a} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

3. Let $\mathbf{a} = \begin{bmatrix} -3 & 2 & x \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$. If $\mathbf{a} \cdot \mathbf{b} = 17$, find x .

4. Let $\mathbf{w} = \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix}$. Compute $\mathbf{w} \cdot \mathbf{w}$.

5. Find all values of x so that $\mathbf{v} \cdot \mathbf{v} = 1$, where $\mathbf{v} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ x \end{bmatrix}$.

6. Let $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$. If $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$, find x and y .

In Exercises 7 and 8, let

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -1 & 5 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 3 & 1 \\ 3 & -4 & 5 \\ 1 & -1 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 & -3 \\ -2 & 1 & 5 \\ 3 & 4 & 2 \end{bmatrix}, \quad \text{and} \quad F = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}.$$

7. If possible, compute:

- (a) AB (b) BA (c) $CB + D$
(d) $AB + DF$ (e) $BA + FD$

8. If possible, compute:

- (a) $A(BD)$ (b) $(AB)D$ (c) $A(C + E)$
(d) $AC + AE$ (e) $(D + F)A$

9. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

Compute the following entries of AB :

- (a) The (1, 2) entry (b) The (2, 3) entry
(c) The (3, 1) entry (d) The (3, 3) entry

10. If $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$, compute DI_2 and I_2D .

11. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}.$$

Show that $AB \neq BA$.

12. If A is the matrix in Example 4 and O is the 3×2 matrix every one of whose entries is zero, compute AO .

In Exercises 13 and 14, let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 3 & -3 & 4 \\ 4 & 2 & 5 & 1 \end{bmatrix}.$$

13. Using the method in Example 12, compute the following columns of AB :

- (a) The first column (b) The third column

14. Using the method in Example 12, compute the following columns of AB :

- (a) The second column (b) The fourth column

15. Let

$$A = \begin{bmatrix} 2 & -3 & 4 \\ -1 & 2 & 3 \\ 5 & -1 & -2 \end{bmatrix} \quad \text{and} \quad c = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}.$$

Express Ac as a linear combination of the columns of A .

16. Let

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 4 & 3 \\ 3 & 0 & -2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 2 & 4 \end{bmatrix}.$$

Express the columns of AB as linear combinations of the columns of A .

17. Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$.

(a) Verify that $AB = 3a_1 + 5a_2 + 2a_3$, where a_j is the j th column of A for $j = 1, 2, 3$.

(b) Verify that $AB = \begin{bmatrix} (\text{row}_1(A))B \\ (\text{row}_2(A))B \end{bmatrix}$.

18. Write the linear combination

$$3 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + 4 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

as a product of a 2×3 matrix and a 3-vector.

19. Consider the following linear system:

$$\begin{array}{rcl} 2x + & & w = 7 \\ 3x + 2y + 3z & = & -2 \\ 2x + 3y - 4z & = & 3 \\ x + & 3z & = 5. \end{array}$$

- (a) Find the coefficient matrix.
(b) Write the linear system in matrix form.
(c) Find the augmented matrix.

20. Write the linear system with augmented matrix

$$\left[\begin{array}{cccc|c} -2 & -1 & 0 & 4 & 5 \\ -3 & 2 & 7 & 8 & 3 \\ 1 & 0 & 0 & 2 & 4 \\ 3 & 0 & 1 & 3 & 6 \end{array} \right].$$

21. Write the linear system with augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 0 & -4 & 3 \\ 0 & 1 & 2 & 5 \\ 1 & 3 & 4 & -1 \end{array} \right].$$

22. Consider the following linear system:

$$\begin{array}{rcl} 3x - y + 2z & = & 4 \\ 2x + y & = & 2 \\ & y + 3z & = 7 \\ 4x & - & z = 4. \end{array}$$

- (a) Find the coefficient matrix.
(b) Write the linear system in matrix form.
(c) Find the augmented matrix.

- (a) How many grams of protein are consumed daily by the males in the project?
- (b) How many grams of fat are consumed daily by the females in the project?

34. (Business) A photography business has a store in each of the following cities: New York, Denver, and Los Angeles. A particular make of camera is available in automatic, semiautomatic, and nonautomatic models. Moreover, each camera has a matched flash unit and a camera is usually sold together with the corresponding flash unit. The selling prices of the cameras and flash units are given (in dollars) by the matrix

$$A = \begin{bmatrix} \text{Auto-} & \text{Semi-} & \text{Non-} \\ \text{matic} & \text{automatic} & \text{automatic} \\ 200 & 150 & 120 \\ 50 & 40 & 25 \end{bmatrix} \begin{matrix} \text{Camera} \\ \text{Flash unit} \end{matrix}$$

The number of sets (camera and flash unit) available at each store is given by the matrix

$$B = \begin{bmatrix} \text{New} & \text{Denver} & \text{Los} \\ \text{York} & & \text{Angeles} \\ 220 & 180 & 100 \\ 300 & 250 & 120 \\ 120 & 320 & 250 \end{bmatrix} \begin{matrix} \text{Automatic} \\ \text{Semiautomatic} \\ \text{Nonautomatic} \end{matrix}$$

- (a) What is the total value of the cameras in New York?
- (b) What is the total value of the flash units in Los Angeles?

35. Let $s_1 = [18.95 \ 14.75 \ 8.98]$ and $s_2 = [17.80 \ 13.50 \ 10.79]$ be 3-vectors denoting the current prices of three items at stores A and B, respectively.

- (a) Obtain a 2×3 matrix representing the combined information about the prices of the three items at the two stores.
- (b) Suppose that each store announces a sale so that the price of each item is reduced by 20%. Obtain a 2×3 matrix representing the sale prices at the two stores.

Exercises 36 through 41 involve bit matrices.

36. For bit vectors $\mathbf{a}$ and $\mathbf{b}$ compute $\mathbf{a} \cdot \mathbf{b}$.

(a) $\mathbf{a} = [1 \ 1 \ 0]$, $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

(b) $\mathbf{a} = [0 \ 1 \ 1 \ 0]$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

37. For bit vectors $\mathbf{a}$ and $\mathbf{b}$ compute $\mathbf{a} \cdot \mathbf{b}$.

(a) $\mathbf{a} = [1 \ 1 \ 0]$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $\mathbf{a} = [1 \ 1]$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

38. Let $\mathbf{a} = [1 \ x \ 0]$ and $\mathbf{b} = \begin{bmatrix} x \\ 1 \\ 1 \end{bmatrix}$ be bit vectors. If $\mathbf{a} \cdot \mathbf{b} = 0$, find all possible values of x .

39. Let $A = \begin{bmatrix} 1 & 1 & x \\ 0 & y & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ be bit matrices. If $AB = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, find x and y .

40. For bit matrices

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

compute AB and BA .

41. For bit matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, determine a 2×2 bit matrix B so that $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Theoretical Exercises

- T.1. Let $\mathbf{x}$ be an n -vector.

- (a) Is it possible for $\mathbf{x} \cdot \mathbf{x}$ to be negative? Explain.
- (b) If $\mathbf{x} \cdot \mathbf{x} = 0$, what is $\mathbf{x}$?

- T.2. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be n -vectors and let k be a real number.

- (a) Show that $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$.
- (b) Show that $(\mathbf{a} + \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot \mathbf{c} + \mathbf{b} \cdot \mathbf{c}$.
- (c) Show that $(k\mathbf{a}) \cdot \mathbf{b} = \mathbf{a} \cdot (k\mathbf{b}) = k(\mathbf{a} \cdot \mathbf{b})$.

- T.3. (a) Show that if A has a row of zeros, then AB has a row of zeros.

- (b) Show that if B has a column of zeros, then AB has a column of zeros.

- T.4. Show that the product of two diagonal matrices is a diagonal matrix.

- T.5. Show that the product of two scalar matrices is a scalar matrix.

- T.6. (a) Show that the product of two upper triangular matrices is upper triangular.

- (b) Show that the product of two lower triangular matrices is lower triangular.

- T.7. Let A and B be $n \times n$ diagonal matrices. Is $AB = BA$? Justify your answer.

- T.8. (a) Let $\mathbf{a}$ be a $1 \times n$ matrix and B an $n \times p$ matrix. Show that the matrix product $\mathbf{a}B$ can be written as

Key Terms

Reduced row echelon form
 Leading one
 Row echelon form
 Elementary row operation
 Row equivalent
 Reduced row echelon form of a matrix

Row echelon form of a matrix
 Gauss-Jordan reduction
 Gaussian elimination
 Back substitution
 Consistent linear system
 Inconsistent linear system

Homogeneous system
 Trivial solution
 Nontrivial solution
 Bit linear systems

1.6 Exercises

In Exercises 1 through 8, determine whether the given matrix is in reduced row echelon form, row echelon form, or neither.

1.
$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

2.
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

3.
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4.
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

5.
$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

6.
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

7.
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

8.
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & -2 & 3 \end{bmatrix}$$

9. Let

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 1 & 4 \\ 4 & 2 & 2 \\ 5 & -1 & 5 \end{bmatrix}$$

Find the matrices obtained by performing the following elementary row operations on A.

(a) Interchanging the second and fourth rows

(b) Multiplying the third row by 3

(c) Adding (-3) times the first row to the fourth row

10. Let

$$A = \begin{bmatrix} 2 & 0 & 4 & 2 \\ 3 & -2 & 5 & 6 \\ -1 & 3 & 1 & 1 \end{bmatrix}$$

Find the matrices obtained by performing the following elementary row operations on A.

(a) Interchanging the second and third rows

(b) Multiplying the second row by (-4)

(c) Adding 2 times the third row to the first row

11. Find three matrices that are row equivalent to

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix}$$

12. Find three matrices that are row equivalent to

$$\begin{bmatrix} 4 & 3 & 7 & 5 \\ -1 & 2 & -1 & 3 \\ 2 & 0 & 1 & 4 \end{bmatrix}$$

In Exercises 13 through 16, find a row echelon form of the given matrix.

13.
$$\begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

14.
$$\begin{bmatrix} 1 & -2 & 0 & 2 \\ 2 & -3 & -1 & 5 \\ 1 & 3 & 2 & 5 \\ 1 & 1 & 0 & 2 \\ 2 & -6 & -2 & 1 \end{bmatrix}$$

15.
$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & 0 & 3 & 4 \\ 0 & 1 & -2 & -1 \\ 2 & 3 & 0 & -3 \end{bmatrix}$$

16.
$$\begin{bmatrix} 2 & -1 & 0 & 1 & 4 \\ 1 & -2 & 1 & 4 & -3 \\ 5 & -4 & 1 & 6 & 5 \\ -7 & 8 & -3 & -14 & 1 \end{bmatrix}$$

17. For each of the matrices in Exercises 13 through 16, find the reduced row echelon form of the given matrix.

18. Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 2 & 1 & -2 \end{bmatrix}.$$

In each part, determine whether $\mathbf{x}$ is a solution to the linear system $A\mathbf{x} = \mathbf{b}$.

(a) $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}; \mathbf{b} = \mathbf{0}$ (b) $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \mathbf{b} = \mathbf{0}$

(c) $\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 3 \\ 6 \\ -5 \end{bmatrix}$

(d) $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

19. Let

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 1 & 3 & 0 & 2 \\ -1 & 2 & 1 & 3 \end{bmatrix}.$$

In each part, determine whether $\mathbf{x}$ is a solution to the homogeneous system $A\mathbf{x} = \mathbf{0}$.

(a) $\mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 5 \\ 2 \end{bmatrix}$ (b) $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

(c) $\mathbf{x} = \begin{bmatrix} 1 \\ -\frac{3}{5} \\ 1 \\ \frac{2}{5} \end{bmatrix}$ (d) $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

In Exercises 20 through 22, find all solutions to the given linear system.

20. (a) $\begin{aligned} x + y + 2z &= -1 \\ x - 2y + z &= -5 \\ 3x + y + z &= 3 \end{aligned}$

(b) $\begin{aligned} x + y + 3z + 2w &= 7 \\ 2x - y + 4w &= 8 \\ 3y + 6z &= 8 \end{aligned}$

(c) $\begin{aligned} x + 2y - 4z &= 3 \\ x - 2y + 3z &= -1 \\ 2x + 3y - z &= 5 \\ 4x + 3y - 2z &= 7 \\ 5x + 2y - 6z &= 7 \end{aligned}$

(d) $\begin{aligned} x + y + z &= 0 \\ x + z &= 0 \\ 2x + y - 2z &= 0 \\ x + 5y + 5z &= 0 \end{aligned}$

21. (a) $\begin{aligned} x + y + 2z + 3w &= 13 \\ x - 2y + z + w &= 8 \\ 3x + y + z - w &= 1 \end{aligned}$

(b) $\begin{aligned} x + y + z &= 1 \\ x + y - 2z &= 3 \\ 2x + y + z &= 2 \end{aligned}$

(c) $\begin{aligned} 2x + y + z - 2w &= 1 \\ 3x - 2y + z - 6w &= -2 \\ x + y - z - w &= -1 \\ 6x + z - 9w &= -2 \\ 5x - y + 2z - 8w &= 3 \end{aligned}$

(d) $\begin{aligned} x + 2y + 3z - w &= 0 \\ 2x + y - z + w &= 3 \\ x - y + w &= -2 \end{aligned}$

22. (a) $\begin{aligned} 2x - y + z &= 3 \\ x - 3y + z &= 4 \\ -5x - 2z &= -5 \end{aligned}$

(b) $\begin{aligned} x + y + z + w &= 6 \\ 2x + y - z &= 3 \\ 3x + y + 2w &= 6 \end{aligned}$

(c) $\begin{aligned} 2x - y + z &= 3 \\ 3x + y - 2z &= -2 \\ x - y + z &= 7 \\ x + 5y + 7z &= 13 \\ x - 7y - 5z &= 12 \end{aligned}$

(d) $\begin{aligned} x + 2y - z &= 0 \\ 2x + y + z &= 0 \\ 5x + 7y + z &= 0 \end{aligned}$

In Exercises 23 through 26, find all values of a for which the resulting linear system has (a) no solution, (b) a unique solution, and (c) infinitely many solutions.

23. $\begin{aligned} x + y - z &= 2 \\ x + 2y + z &= 3 \\ x + y + (a^2 - 5)z &= a \end{aligned}$

24. $\begin{aligned} x + y + z &= 2 \\ 2x + 3y + 2z &= 5 \\ 2x + 3y + (a^2 - 1)z &= a + 1 \end{aligned}$

25. $\begin{aligned} x + y + z &= 2 \\ x + 2y + z &= 3 \\ x + y + (a^2 - 5)z &= a \end{aligned}$

26. $\begin{aligned} x + y &= 3 \\ x + (a^2 - 8)y &= a \end{aligned}$

In Exercises 27 through 30, solve the linear system with the given augmented matrix.

27. (a) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 \end{array} \right]$

$$(b) \begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{bmatrix}$$

$$28. (a) \begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 5 & 7 & 9 & 0 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 2 & 1 & 7 \\ 2 & 0 & 1 & 4 \\ 1 & 0 & 2 & 5 \\ 1 & 2 & 3 & 11 \\ 2 & 1 & 4 & 12 \end{bmatrix}$$

$$29. (a) \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & -2 & 3 & 4 \\ 2 & -1 & -3 & 5 \\ 3 & 0 & 1 & 2 \\ 3 & -3 & 0 & 7 \end{bmatrix}$$

$$30. (a) \begin{bmatrix} 4 & 2 & -1 & 5 \\ 3 & 3 & 6 & 1 \\ 5 & 1 & -8 & 8 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{bmatrix}$$

31. Let $f: R^3 \rightarrow R^3$ be the matrix transformation defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Find x, y, z so that $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$.

32. Let $f: R^3 \rightarrow R^3$ be the matrix transformation defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Find x, y, z so that $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$.

33. Let $f: R^3 \rightarrow R^3$ be the matrix transformation defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Find an equation relating a, b , and c so that we can

always compute values of x, y , and z for which

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

34. Let $f: R^3 \rightarrow R^3$ be the matrix transformation defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Find an equation relating a, b , and c so that we can always compute values of x, y , and z for which

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

In Exercises 35 and 36, solve the linear systems $A\mathbf{x} = \mathbf{b}_1$ and $A\mathbf{x} = \mathbf{b}_2$ separately and then by obtaining the reduced row echelon form of the augmented matrix $[A \mid \mathbf{b}_1 \mid \mathbf{b}_2]$. Compare your answers.

$$35. A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \mathbf{b}_1 = \begin{bmatrix} 1 \\ -8 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

$$36. A = \begin{bmatrix} 1 & -2 & 0 \\ -3 & 2 & -1 \\ 4 & -2 & 3 \end{bmatrix}, \mathbf{b}_1 = \begin{bmatrix} 3 \\ -7 \\ 12 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -4 \\ 6 \\ -10 \end{bmatrix}$$

In Exercises 37 and 38, let

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix}$$

37. Find a nontrivial solution to the homogeneous system $(-4I_3 - A)\mathbf{x} = \mathbf{0}$.*

38. Find a nontrivial solution to the homogeneous system $(2I_3 - A)\mathbf{x} = \mathbf{0}$.*

39. Find an equation relating a, b , and c so that the linear system

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 3y + 3z &= b \\ 5x + 9y - 6z &= c \end{aligned}$$

is consistent for any values of a, b , and c that satisfy that equation.

40. Find an equation relating a, b , and c so that the linear system

$$\begin{aligned} 2x + 2y + 3z &= a \\ 3x - y + 5z &= b \\ x - 3y + 2z &= c \end{aligned}$$

is consistent for any values of a, b , and c that satisfy that equation.

*This type of problem will play a key role in Chapter 8.