

Name _____

The following is a selection of practice problems for test 2. This is not the complete list of problems that are possible to appear in the test. Please see MML HW and the suggested exercises list on your syllabus for the complete list.

All answers must be left in exact and simplified form as in class.

1. Find the exact value of each expression in radians. If there is no value, write undefined.

a) $\cos^{-1} 0$ b) $\sin^{-1}(-1)$ c) $\tan^{-1}(-1)$ d) $\sin^{-1}\left(\frac{3}{2}\right)$

e) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ f) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ g) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

2. Find the exact value of each expression. Do not use a calculator.

a) $\cos^{-1}\left[\cos\left(\frac{7\pi}{9}\right)\right]$ b) $\tan^{-1}\left[\tan\left(-\frac{4\pi}{19}\right)\right]$ c) $\sin^{-1}\left[\sin\left(\frac{7\pi}{8}\right)\right]$ d) $\tan^{-1}\left[\tan\left(\frac{9\pi}{10}\right)\right]$

3. Find the exact value of each expression. Do not use a calculator.

a) $\cos\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$ b) $\tan\left[\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$ c) $\sec\left[\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$ d) $\sec\left[\tan^{-1}\left(\frac{9}{10}\right)\right]$

4. Find the exact value of each expression. Do not use a calculator.

a) $\cos^{-1}\left[\sin\left(\frac{11\pi}{6}\right)\right]$ b) $\sin^{-1}\left[\cos\left(-\frac{5\pi}{6}\right)\right]$ c) $\sin^{-1}\left[\cos\left(\frac{2\pi}{3}\right)\right]$

5. Write the trigonometric expression as an algebraic expression of u.

a) $\sec(\sin^{-1} u)$ b) $\cot(\tan^{-1} u)$ c) $\sin(\cos^{-1} u)$

6. Solve each of the following equations on the interval $0 \leq \theta < 2\pi$.

a) $2 \sin \theta - \sqrt{2} = 0$ b) $\tan \theta - \sqrt{3} = 0$ c) $7 \sec \theta + 9 = -5$ d) $3\sqrt{2} \cos \theta + 1 = -2$

7. Solve each of the following equations on the interval $0 \leq \theta < 2\pi$.

a) $4 \cos^2 \theta = 1$ b) $8 \sin^2 \theta - 6 = 0$

8. Solve each of the following equations on the interval $0 \leq \theta < 2\pi$.

a) $\sqrt{2} \cos^2 \theta + \cos \theta = 0$ b) $2 \sin^2 \theta + \sin \theta - 1 = 0$ c) $(\cot \theta - 1)(\csc \theta - 1) = 0$

d) $\cos^2 \theta - \sin^2 \theta = 1 - \sin \theta$ e) $4 \sin^2 \theta + 13 \sin \theta + 9 = 0$ f) $1 - \sin \theta = 2 \cos^2 \theta$

9. Establish the identity.

a) $\cos \theta \tan \theta = \sin \theta$ b) $\cos \theta (\cot \theta + \tan \theta) = \csc \theta$ c) $\cot \theta \tan \theta - \sin^2 \theta = \cos^2 \theta$

d) $(\sec \theta + 1)(\sec \theta - 1) = \tan^2 \theta$ e) $\csc^2 \theta (1 - \cos^2 \theta) = 1$ f) $1 - \frac{\sin^2 \theta}{1 + \cos \theta} = \cos \theta$

10. Use a sum or difference formula to find the exact value of the trigonometric function.

a) $\sin 105^\circ$ b) $\tan 15^\circ$ c) $\sin \frac{\pi}{12}$ d) $\cos \frac{7\pi}{12}$ e) $\sin \frac{13\pi}{12}$

11. Find the exact value of each expression.

a) $\sin \frac{5\pi}{12} \cos \frac{3\pi}{4} - \cos \frac{5\pi}{12} \sin \frac{3\pi}{4}$ b) $\sin 25^\circ \cos 20^\circ + \cos 25^\circ \sin 20^\circ$

c) $\cos 35^\circ \cos 10^\circ - \sin 35^\circ \sin 10^\circ$

12. Let $\sin \alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$ and $\cos \beta = \frac{\sqrt{3}}{4}$, $-\frac{\pi}{2} < \beta < 0$. Find the exact value of the following.

a) $\sin(\alpha + \beta)$ b) $\cos(\alpha + \beta)$ c) $\tan(\alpha - \beta)$

13. Let $\tan \alpha = -\frac{4}{3}$, $\frac{\pi}{2} < \alpha < \pi$ and $\cos \beta = \frac{1}{2}$, $0 < \beta < \frac{\pi}{2}$. Find the exact value of the following.

a) $\sin(\alpha - \beta)$ b) $\cos(\alpha - \beta)$

14. If $\sin \theta = \frac{4}{9}$, $\frac{\pi}{2} < \theta < \pi$, find the exact value of the following.

a) $\cos \theta$ b) $\sin\left(\theta + \frac{\pi}{6}\right)$ c) $\cos\left(\theta - \frac{\pi}{3}\right)$ d) $\tan\left(\theta + \frac{\pi}{4}\right)$

15. For each of the following establish the given identity.

a) $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha \cos\beta$ b) $\frac{\cos(\alpha+\beta)}{\sin\alpha \cos\beta} = \cot\alpha - \tan\beta$

16. Find the exact value of each expression.

a) $\sin\left(\sin^{-1} 0 - \cos^{-1} \frac{1}{2}\right)$ b) $\cos\left[\sin^{-1} \frac{5}{13} - \cos^{-1}\left(-\frac{8}{17}\right)\right]$

17. If $\sin \theta = \frac{1}{6}$, $0 < \theta < \frac{\pi}{2}$, find the exact value of the following.

a) $\sin(2\theta)$ b) $\cos(2\theta)$ c) $\sin \frac{\theta}{2}$ d) $\cos \frac{\theta}{2}$

18. Use the half-angle formulas to find the exact value of the each of the following expressions.

a) $\sin 15^\circ$ b) $\tan 165^\circ$ c) $\tan \frac{5\pi}{8}$

19. Find the exact value of each of the following expression.

a) $\sin\left[2 \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$ b) $\cos\left(2 \sin^{-1} \frac{1}{2}\right)$ c) $\tan\left(2 \sin^{-1} \frac{3}{5}\right)$

Answers

1. a) $\frac{\pi}{2}$ b) $-\frac{\pi}{2}$ c) $-\frac{\pi}{4}$ d) *undefined*

e) $\frac{\pi}{4}$ f) $\frac{\pi}{6}$ g) $\frac{3\pi}{4}$

2. a) $\frac{7\pi}{9}$ b) $-\frac{4\pi}{19}$ c) $\frac{\pi}{8}$ d) $-\frac{\pi}{10}$

3. a) $\frac{\sqrt{2}}{2}$ b) -1 c) $\frac{2}{\sqrt{2}} = \sqrt{2}$ d) $\frac{\sqrt{181}}{10}$

4. a) $\cos^{-1}\left[-\frac{1}{2}\right] = \frac{2\pi}{3}$ b) $\sin^{-1}\left[-\frac{\sqrt{3}}{2}\right] = -\frac{\pi}{3}$ c) $\sin^{-1}\left[-\frac{1}{2}\right] = -\frac{\pi}{6}$

5. a) $\frac{1}{\sqrt{1-u^2}}$ b) $\frac{1}{u}$ c) $\sqrt{1-u^2}$

6. a) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ b) $\left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$ c) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ d) $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$

7. a) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ b) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

8. a) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}\right\}$ b) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ c) $\left\{\frac{\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{2}\right\}$

d) $\left\{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$ e) $\left\{\frac{3\pi}{2}\right\}$ f) $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}\right\}$

9.

10. a) $\frac{\sqrt{2}+\sqrt{6}}{4}$ b) $\frac{3-\sqrt{3}}{3+\sqrt{3}}$ c) $\frac{\sqrt{6}-\sqrt{2}}{4}$ d) $\frac{\sqrt{2}-\sqrt{6}}{4}$ e) $\frac{\sqrt{2}-\sqrt{6}}{4}$

11. a) $-\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{2}}{2}$ c) $\frac{\sqrt{2}}{2}$

12. a) $\frac{3\sqrt{3}-4\sqrt{13}}{20}$ b) $\frac{4\sqrt{3}+3\sqrt{13}}{20}$ c) $\frac{\frac{3}{4}-\left(-\frac{\sqrt{13}}{\sqrt{3}}\right)}{1+\left(\frac{3}{4}\right)\left(-\frac{\sqrt{13}}{\sqrt{3}}\right)} = \frac{3\sqrt{3}+4\sqrt{13}}{4\sqrt{3}-3\sqrt{13}}$

13. a) $\frac{4+3\sqrt{3}}{10}$ b) $\frac{-3+4\sqrt{3}}{10}$

14. a) $-\frac{\sqrt{65}}{9}$ b) $\frac{4\sqrt{3}-\sqrt{65}}{18}$ c) $\frac{-\sqrt{65}+4\sqrt{3}}{18}$ d) $\frac{\frac{-4}{\sqrt{65}}+1}{1+\frac{4}{\sqrt{65}}} = \frac{-4+\sqrt{65}}{\sqrt{65}+4}$

15.

16. a) $-\frac{\sqrt{3}}{2}$ b) $-\frac{21}{221}$

Answers

$$17. \text{ a) } \frac{\sqrt{35}}{18} \quad \text{ b) } \frac{17}{18} \quad \text{ c) } \sqrt{\frac{1-\frac{\sqrt{35}}{6}}{2}} = \sqrt{\frac{6-\sqrt{35}}{12}} \quad \text{ d) } \sqrt{\frac{1+\frac{\sqrt{35}}{6}}{2}} = \sqrt{\frac{6+\sqrt{35}}{12}}$$

$$18. \text{ a) } \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2} \quad \text{ b) } \frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -2 + \sqrt{3} \quad \text{ c) } \frac{1+\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -\frac{2+\sqrt{2}}{\sqrt{2}}$$

$$19. \text{ a) } \frac{\sqrt{3}}{2} \quad \text{ b) } \frac{1}{2} \quad \text{ c) } \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{24}{7}$$