0.1 Basic Differentiation Rules

The following rules allow us the find the derivatives of functions that can be created by combining familiar functions.

Constant Multiple Rule

$$[cf(x)]' = cf'(x)$$

In words, the derivative of a constant times a function is the constant times the derivative of the function.

Examples of the Constant Multiple Rule

CM 1. If $f(x) = 5x$ then we use the constant multiple rule with $c = 5$ and we get $f'(x) = 5(1) = 5$.

CM 2. If $f(x) = \frac{x^3}{9}$ then we use the constant multiple rule with $c = \frac{1}{9}$ and we get $f'(x) = \frac{1}{9} \cdot 3x^2 = \frac{x^2}{3}$.

CMR 3. If $f(x) = \frac{4}{x^2}$ then we rewrite $f(x)$ as $4x^{-2}$ and we use the constant multiple rule with $c = 4$, giving $f'(x) = 4(-2x^{-3}) = -8x^{-3} = -\frac{8}{x^3}$.

CM 4. If $f(x) = -3\cos(x)$ then we use the constant multiple rule with $c = -3$ and we get $f'(x) = -3(-\sin(x)) = 3\sin(x)$.

CM 5. If $f(x) = \pi \tan^{-1}(x)$ then we use the constant multiple rule with $c = \pi$ and we get $f'(x) = \pi\frac{1}{1+x^2} = \frac{\pi}{1+x^2}$.

CM 6. If $f(x) = \frac{2e^x}{5}$ then we use the constant multiple rule with $c = \frac{2}{5}$ and we get $f'(x) = \frac{2}{5}e^x = \frac{2e^x}{5}$.

CM 7. If $f(x) = \frac{\ln(x)}{\ln(4)}$ then we use the constant multiple rule with $c = \frac{1}{\ln(4)}$ and we get $f'(x) = \frac{1}{\ln(4)} \cdot \frac{1}{x} = \frac{1}{x\ln(4)}$.

Sum Rule

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

In words, the derivative of a sum is the sum of the derivatives.

Examples of the Sum Rule

SR 1. If $f(x) = x^3 + x^2$ then $f'(x) = 3x^2 + 2x$.

SR 2. If $f(x) = \frac{1}{x} + \frac{1}{x^2}$ then we rewrite $f(x)$ as $x^{-1} + x^{-2}$ and

$$f'(x) = -x^{-2} - 2x^{-3} = -\frac{1}{x^2} - \frac{2}{x^3} = -\frac{x+2}{x^3}.$$

SR 3. If $f(x) = \sqrt{x} + \sqrt[3]{x}$ then we rewrite $f(x)$ as $x^{1/2} + x^{1/3}$ and

$$\begin{aligned} f'(x) &= \tfrac{1}{2}x^{-1/2} + \tfrac{1}{3}x^{-2/3} \\ &= \tfrac{1}{2} \cdot \frac{1}{x^{1/2}} + \tfrac{1}{3} \cdot \frac{1}{x^{2/3}} \\ &= \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}}. \end{aligned}$$

SR 4. If $f(x) = 4\sin(x) + 5\cos(x)$ then

$$f'(x) = 4\cos(x) - 5\sin(x).$$

SR 5. If $f(x) = 2^x + 3^x + 5^x$ then $f'(x) = 2^x\ln(2) + 3^x\ln(3) + 5^x\ln(5)$.

SR 6. If $f(x) = \ln(x) + \tan^{-1}(x)$ then

$$f'(x) = \frac{1}{x} + \frac{1}{1+x^2} = \frac{x^2+x+1}{x^3+x}.$$

Difference Rule

$$[f(x) - g(x)]' = f'(x) - g'(x)$$

In words, the derivative of a difference is the difference of the derivatives.

Examples of the Difference Rule

DR 1. If $f(x) = x^5 - x$ then $f'(x) = 5x^4 - 1$.

DR 2. If $f(x) = \frac{1}{x^3} - \frac{1}{x^4}$ then we rewrite $f(x)$ as $x^{-3} - x^{-4}$ and $f'(x) = -3x^{-4} + 4x^{-5} = -\frac{3}{x^4} + \frac{4}{x^5} = \frac{4}{x^5} - \frac{3}{x^4} = \frac{4x^4 - 3x^5}{x^9} = \frac{4-3x}{x^5}$.

DR 3. If $f(x) = \sqrt{x} - \sqrt[4]{x^3}$ then we rewrite $f(x)$ as $x^{1/2} + x^{3/4}$ and $f'(x) = \frac{1}{2}x^{-1/2} - \frac{3}{4}x^{-1/4} = \frac{1}{2} \cdot \frac{1}{x^{1/2}} - \frac{3}{4} \cdot \frac{1}{x^{1/4}} = \frac{1}{2\sqrt{x}} - \frac{3}{4\sqrt[4]{x}}$.

DR 4. If $f(x) = 2\sin(x) - 3\cos(x)$ then $f'(x) = 2\cos(x) - (-3\sin(x)) = 2\cos(x) + 3\sin(x)$.

DR 5. If $f(x) = e^{x+1} - e^{x+2}$ then we rewrite $f(x)$ as $e \cdot e^x - e^2 \cdot e^x$ and we use the constant multiple rule with constants e and e^2 together with the difference rule to get $f'(x) = e \cdot e^x - e^2 \cdot e^x = e^{x+1} - e^{x+2}$.

DR 6. If $f(x) = \log(x) - \sin^{-1}(x)$ then $f'(x) = \frac{1}{x\ln(10)} - \frac{1}{\sqrt{1-x^2}}$.