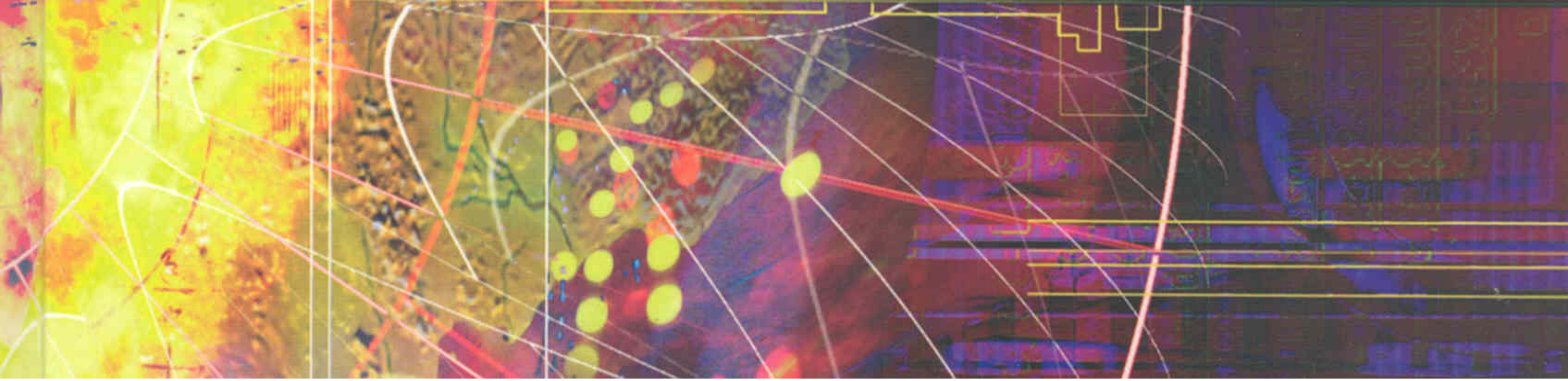
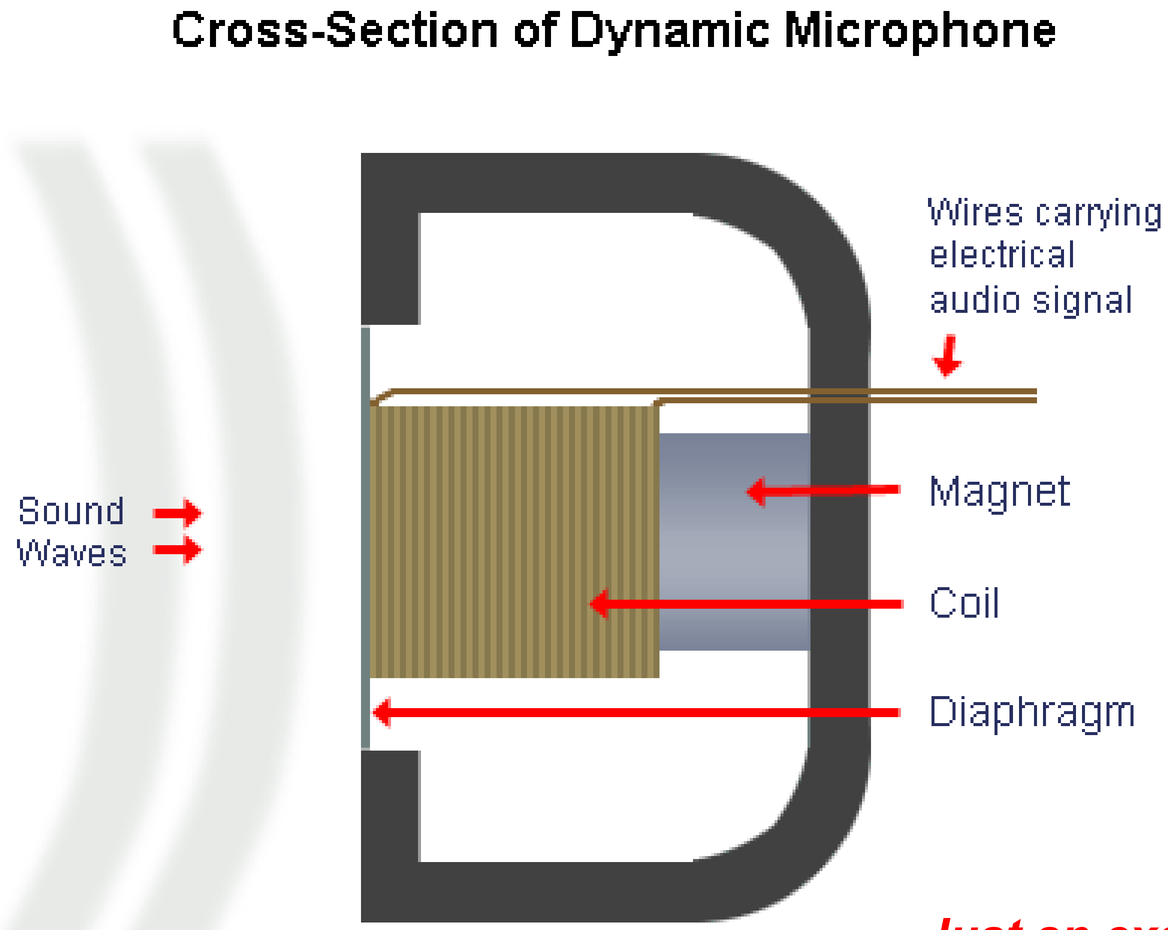


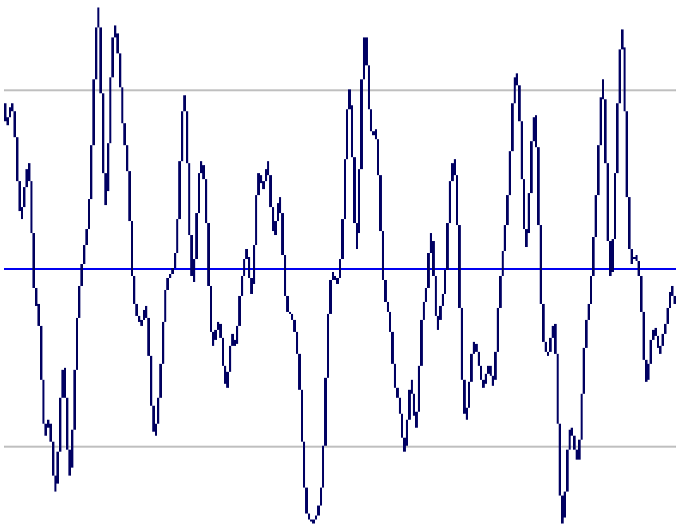
Analog to Digital Conversion

CSIS Faculty Seminar Series #1
Numbers & Bytes Meeting
04 February 2011 : 4:00 – 5:00 PM
CSCC Room 203

William M. Jones, Ph.D.

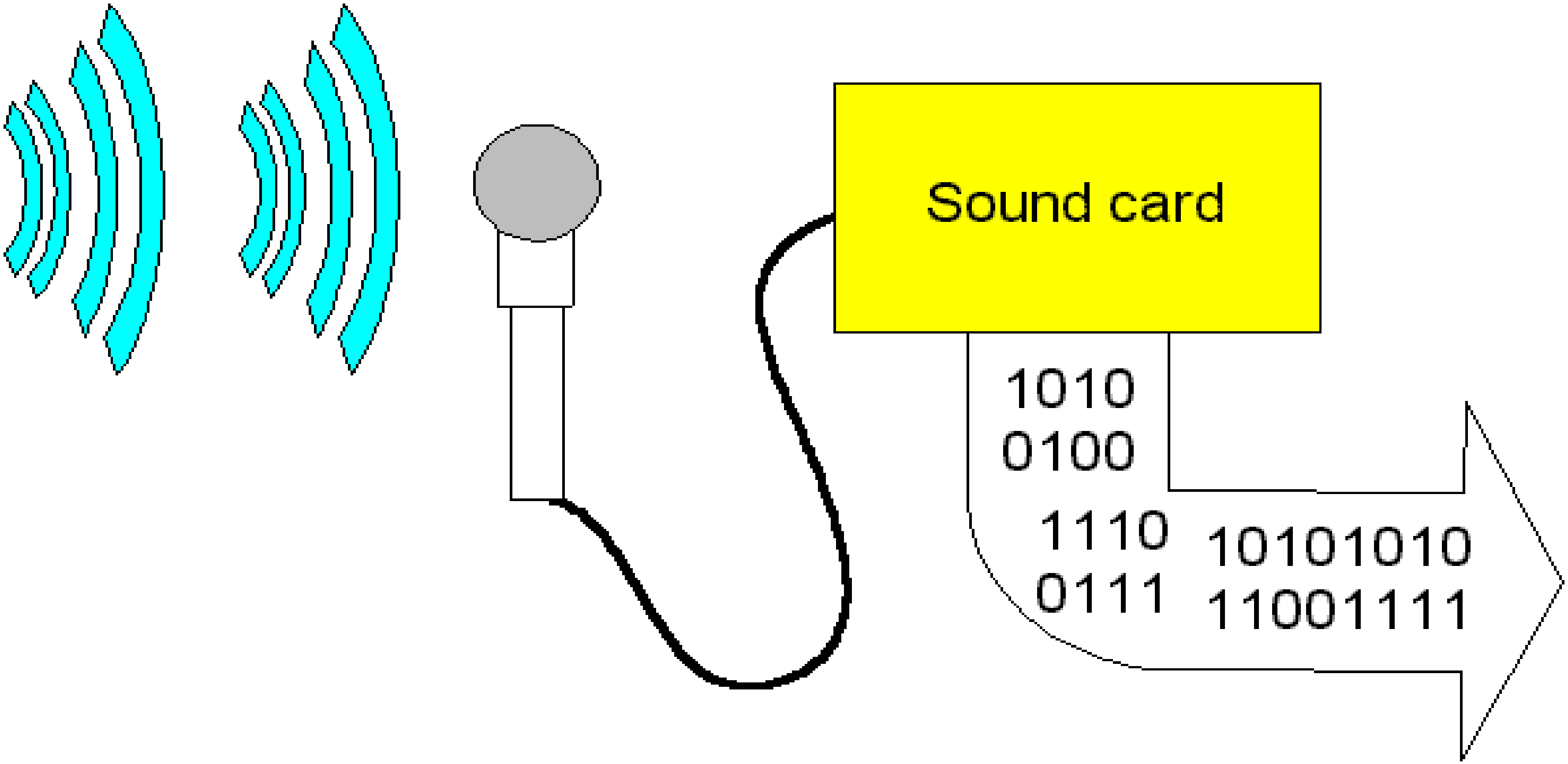
What are analog signals? WDTCF?





Just an example, many, many more …..
Sensors, transducers, etc.

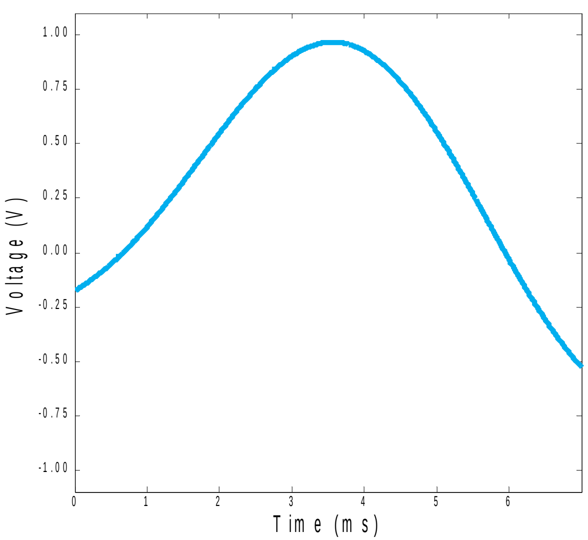
So what's this talk about?



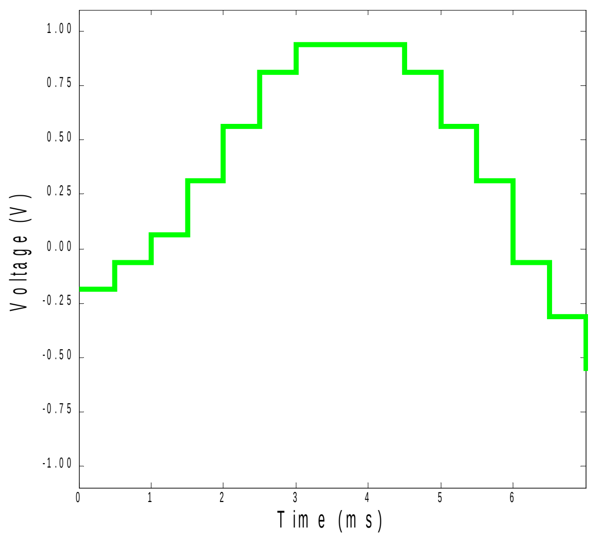
What's happening in that sound card?

Conversion from analog to digital

- We will consider the problem of converting an analog waveform into binary values and then converting it back into analog.



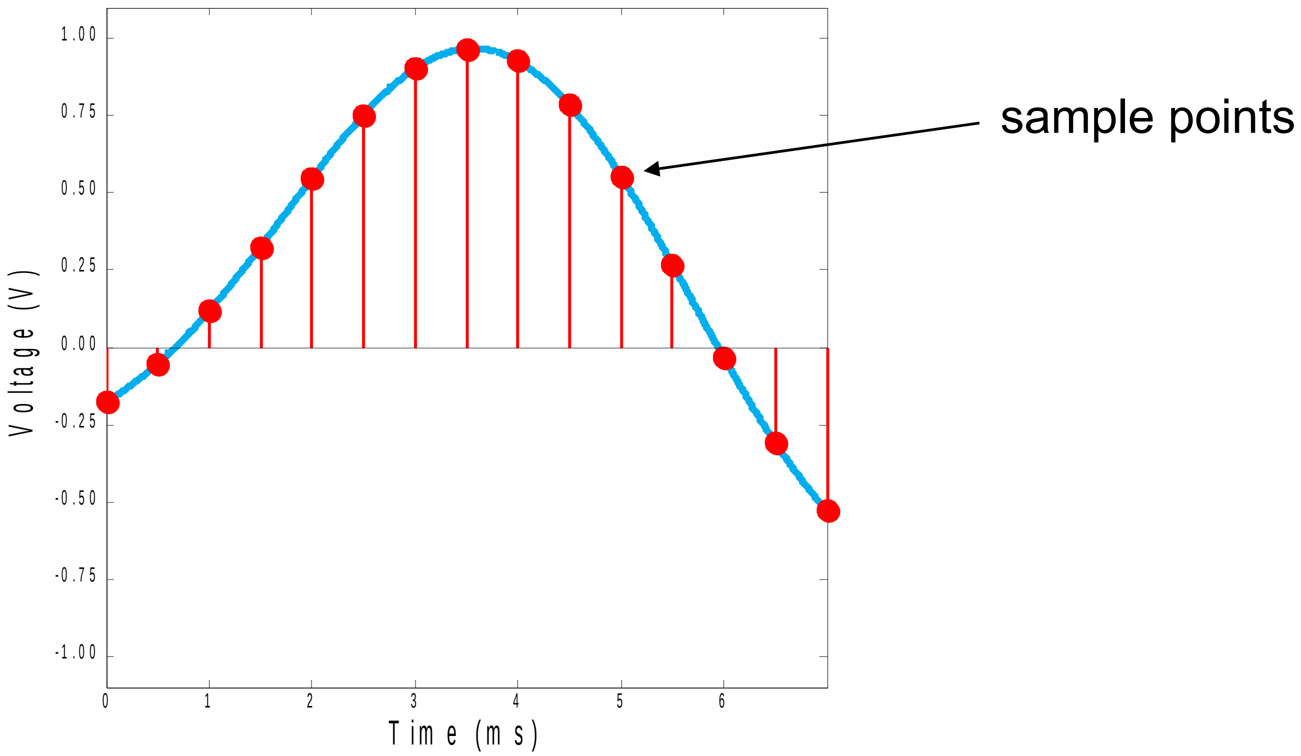
0110
0111
1000
1010
1100
1110
1111
1111
1111
1110
1100
1010
0111
0101
0011



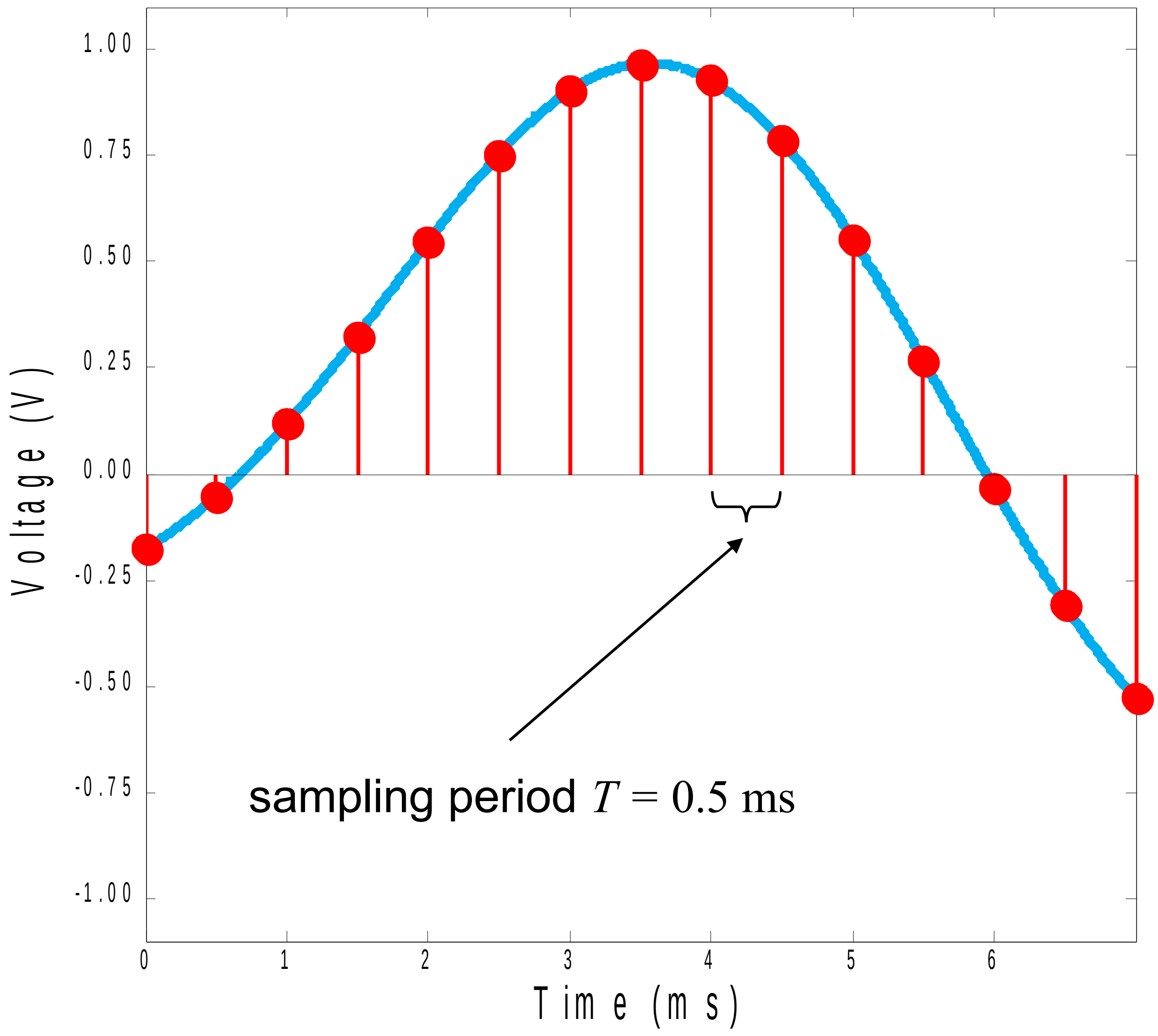
analog → digital → analog

Sampling

- The analog waveform is composed of an infinite number of points.
- Therefore, we must take samples of this continuous waveform to send.



Sampling



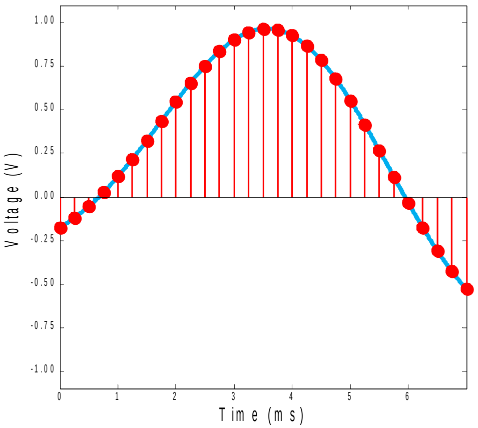
sampling frequency f

$$f = \frac{1}{T} = \frac{1}{0.0005} = 2 \text{ kHz}$$

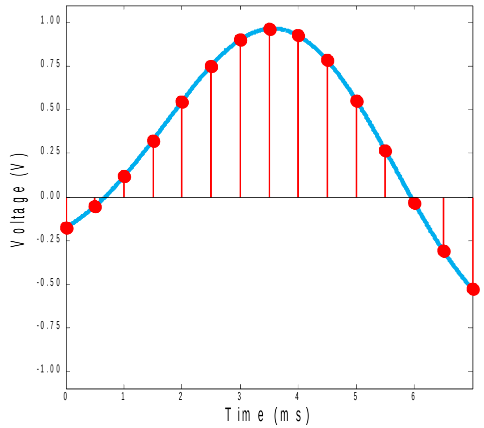
Sampling

- How fast does our sampling rate f need to be?

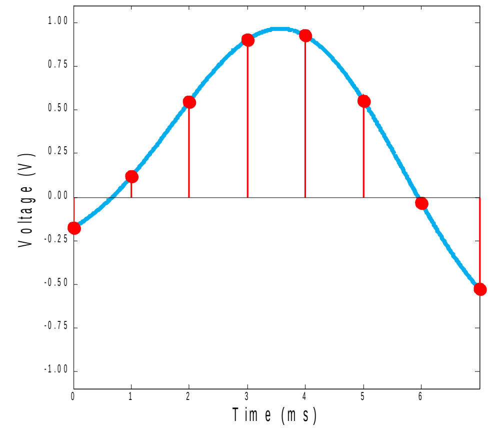
sampling frequency $f = 4$ kHz



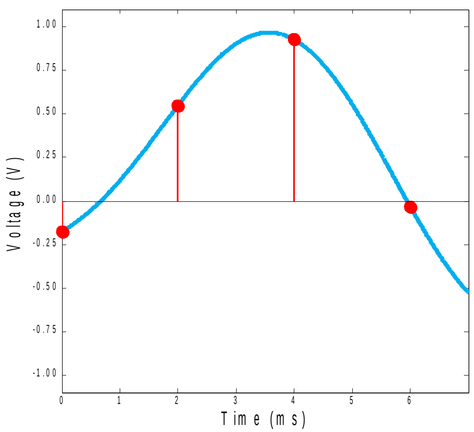
sampling frequency $f = 2$ kHz



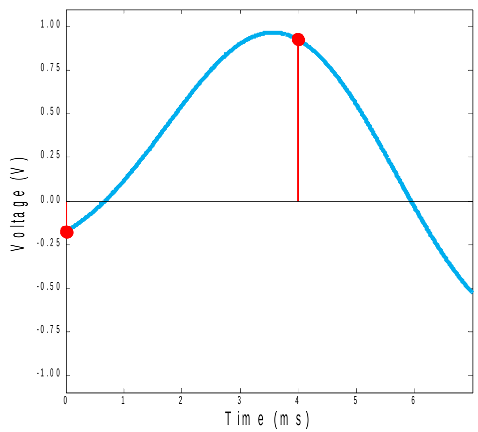
sampling frequency $f = 1$ kHz



sampling frequency $f = 500$ Hz



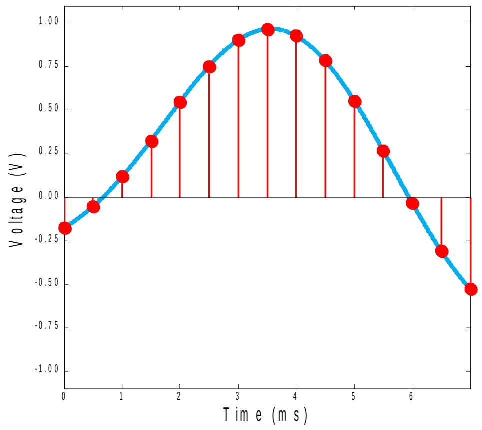
sampling frequency $f = 250$ Hz



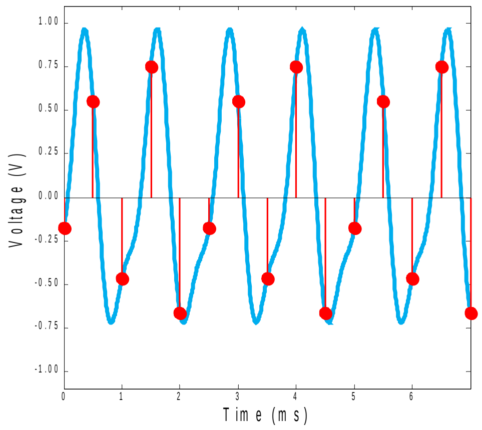
Sampling

- The number of samples required is dictated by the frequency content of our analog waveform.
  - A slowly changing waveform (i.e. low frequency) can be sampled at a lower rate.
  - A rapidly changing waveform (i.e. high frequency) must be sampled at a high rate in order to capture the rapid changes.

sampling frequency $f = 2$ kHz



sampling frequency $f = 2$ kHz



Minimum sampling frequency

- The minimum sampling rate required in order to accurately reconstruct the analog input is given by the Nyquist sampling rate f_N given

$$f_N \geq 2 f_m$$

- where f_m is the highest frequency of the analog signal.
  - The Nyquist rate is a theoretical minimum.
  - In practice, sampling rates are typically 2.5 to 3 times the Nyquist rate f_N.
  - Audio CDs ?

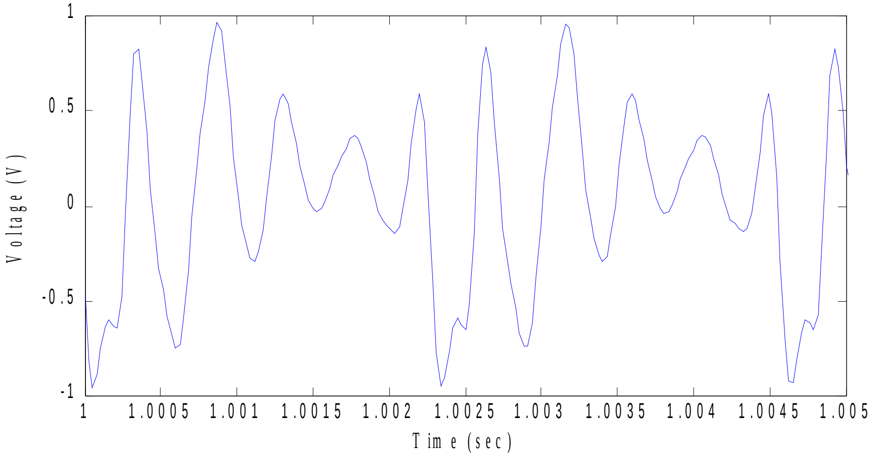
Example Problem 1

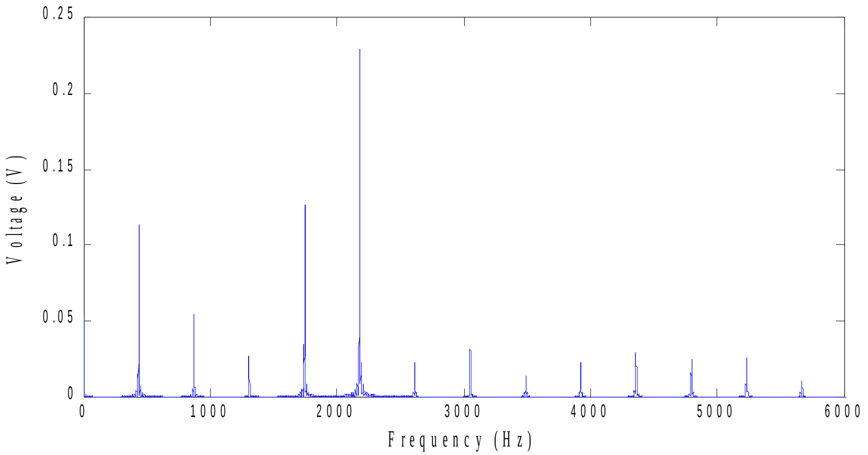
Consider the signal from the oboe depicted below in time and frequency domain representations.

a. What is the maximum frequency present in the oboe signal?

b. Based upon this, what would be the minimum sampling rate according to Nyquist?

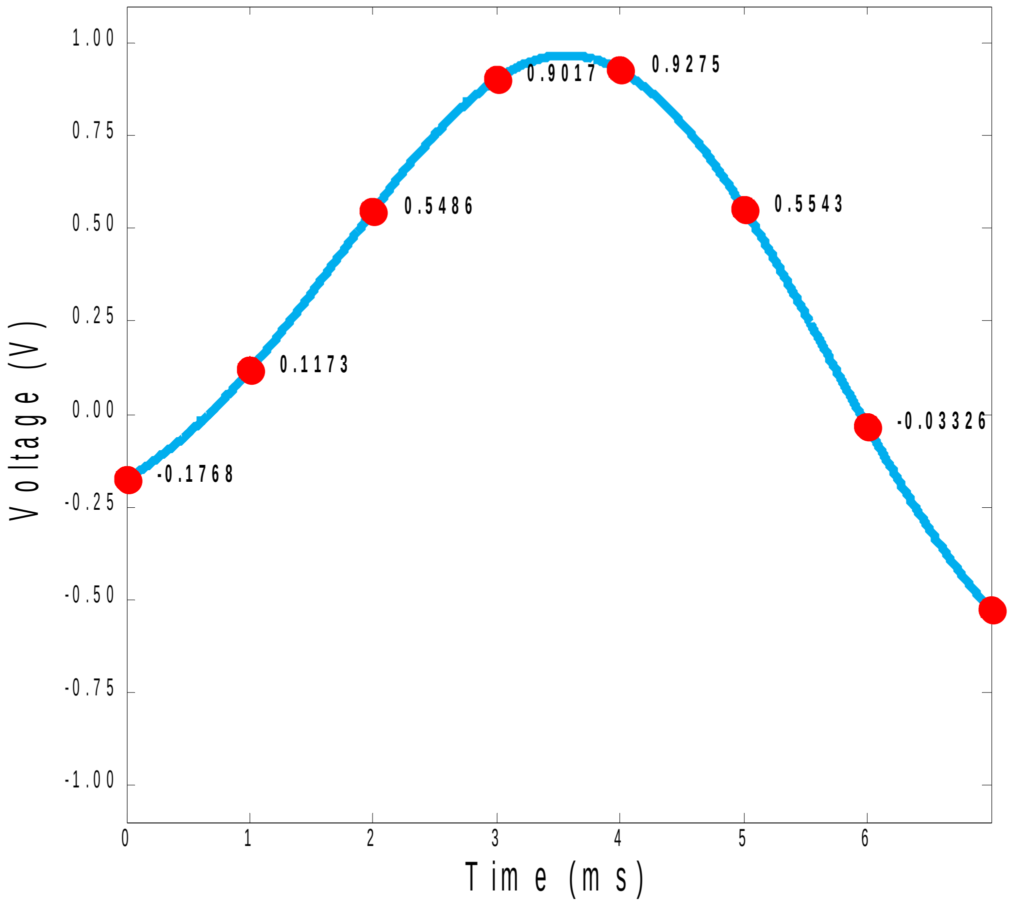
c. What would be a practical sampling rate?





Sampled waveform

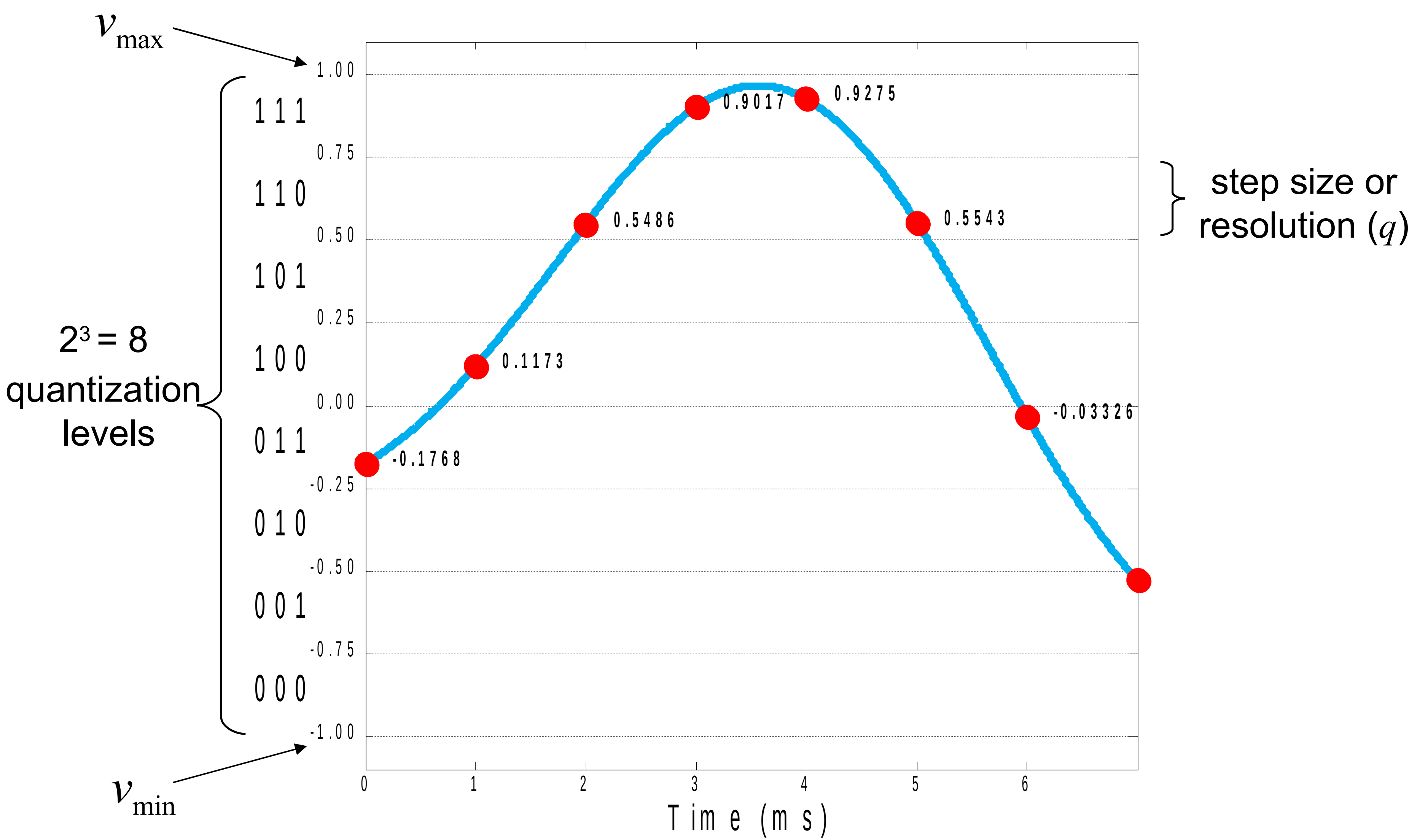
- We can now determine the amplitudes associated with each sample point.



Quantization

- We now need to convert these amplitudes (real numbers) into binary integers.
- The process of mapping the sampled analog voltage levels to discrete, binary values is called **quantization**.
- Quantizers are characterized length of the binary words they produce.
- An N-bit quantizer has 2^N levels and outputs binary numbers of length N.
  - Telephones use 8-bit encoding $\rightarrow 2^8 = 256$ levels
  - CD audio use 16-bit encoding $\rightarrow 2^{16} = 65{,}536$ levels

Quantization intervals (3-bit)



Quantization intervals

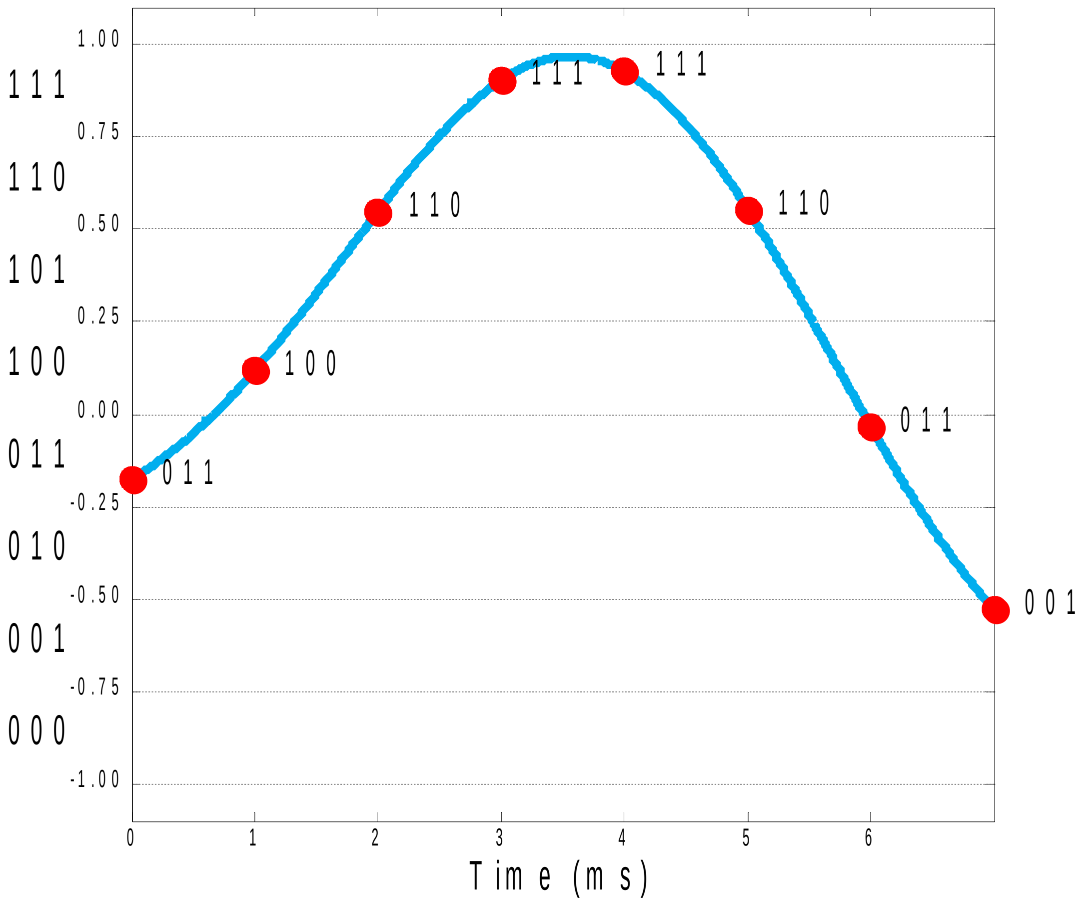
- Quantizers are limited to specific voltage range.
  - For this example we will assume that our analog input falls within a range of -1.0 to +1.0 volts.
- The quantizer will partition this range into 2^N steps of size q given

$$q = \frac{v_{\max} - v_{\min}}{2^N}$$ quantizer step size [volts]

  - q is also called the resolution. *What improves resolution?*
- Each of these intervals (or bins) is assigned a binary value from 0 to $2^N - 1$.

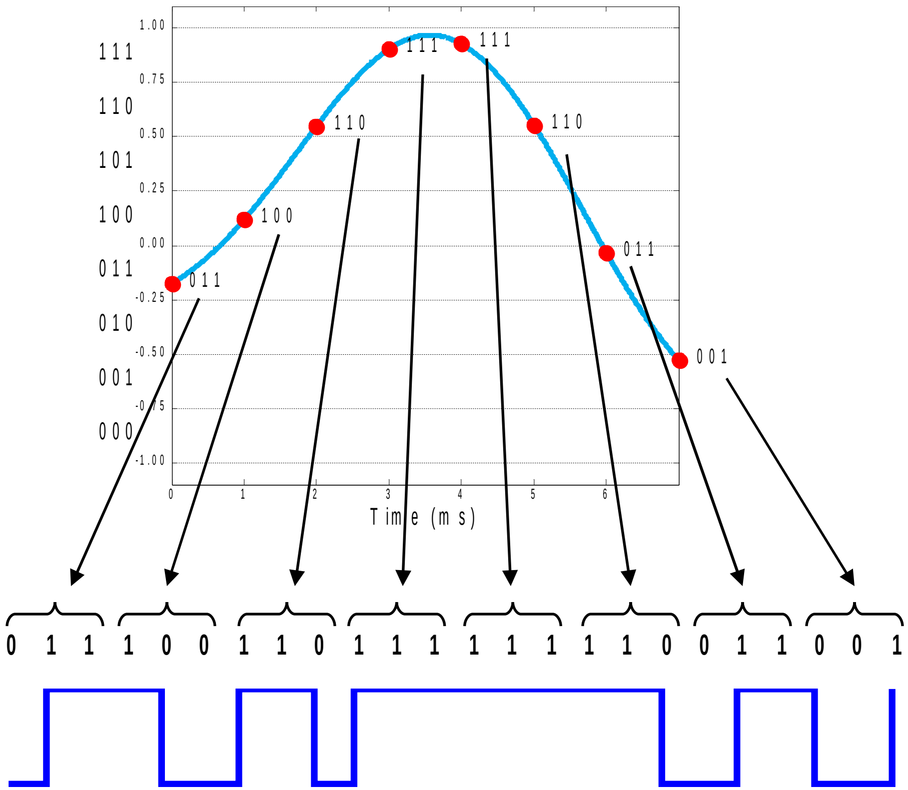
Quantization intervals

- If sampled point falls within that interval (or bin), it is assigned that binary value.



Digital signal

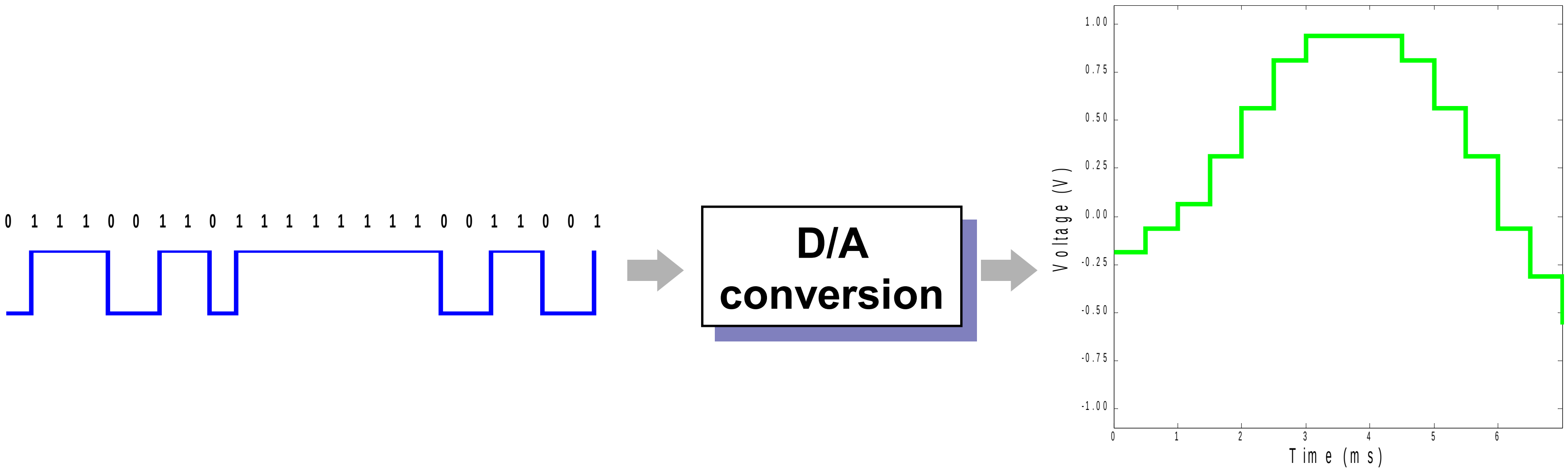
- These binary values are then transmitted to the receiver as a digital signal.



transmitted digital signal

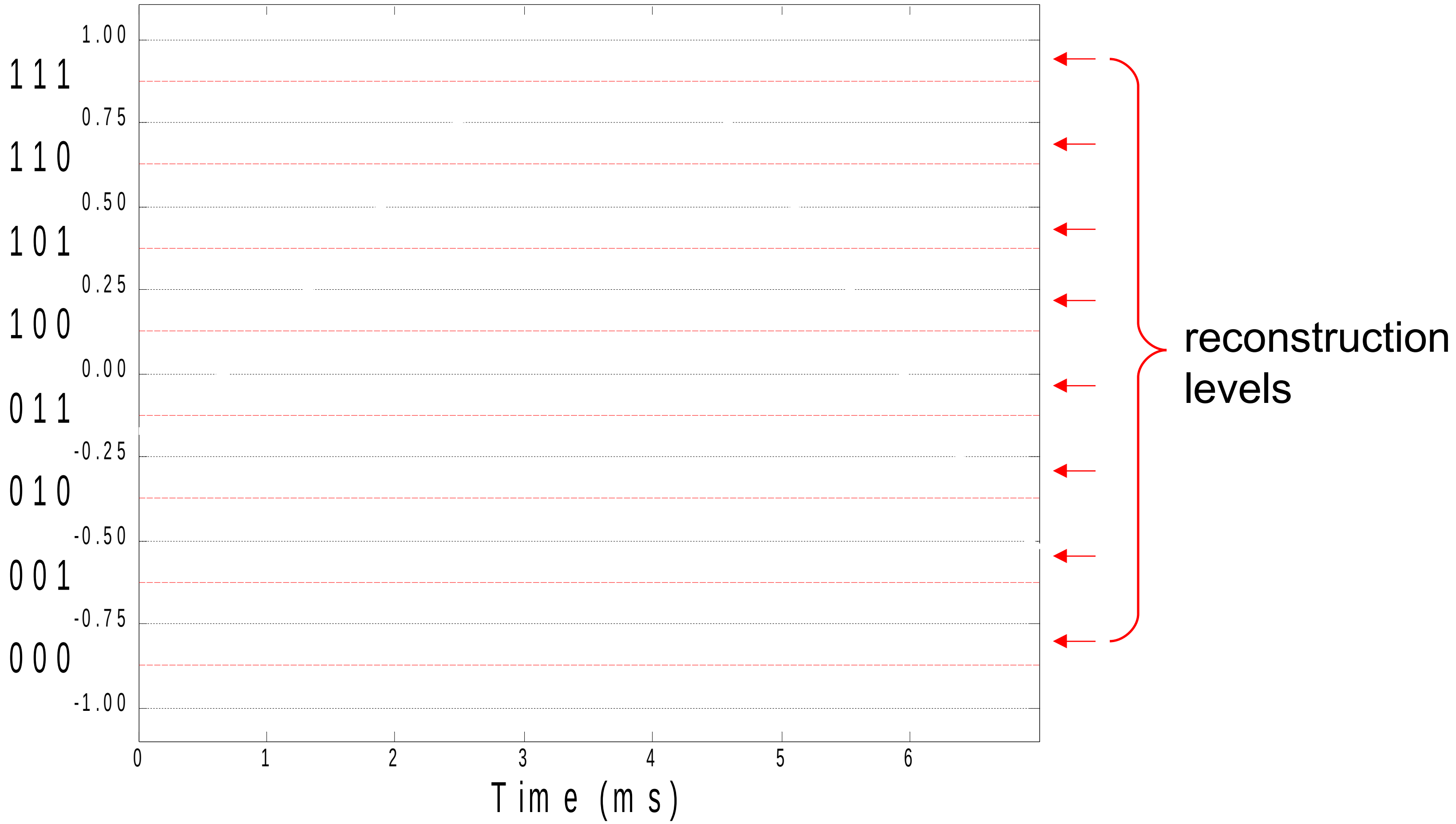
Digital-to-analog (D/A) conversion

- At the receiver, these binary values must be converted back into an analog signal.
- This process is called digital-to-analog (D/A) conversion.

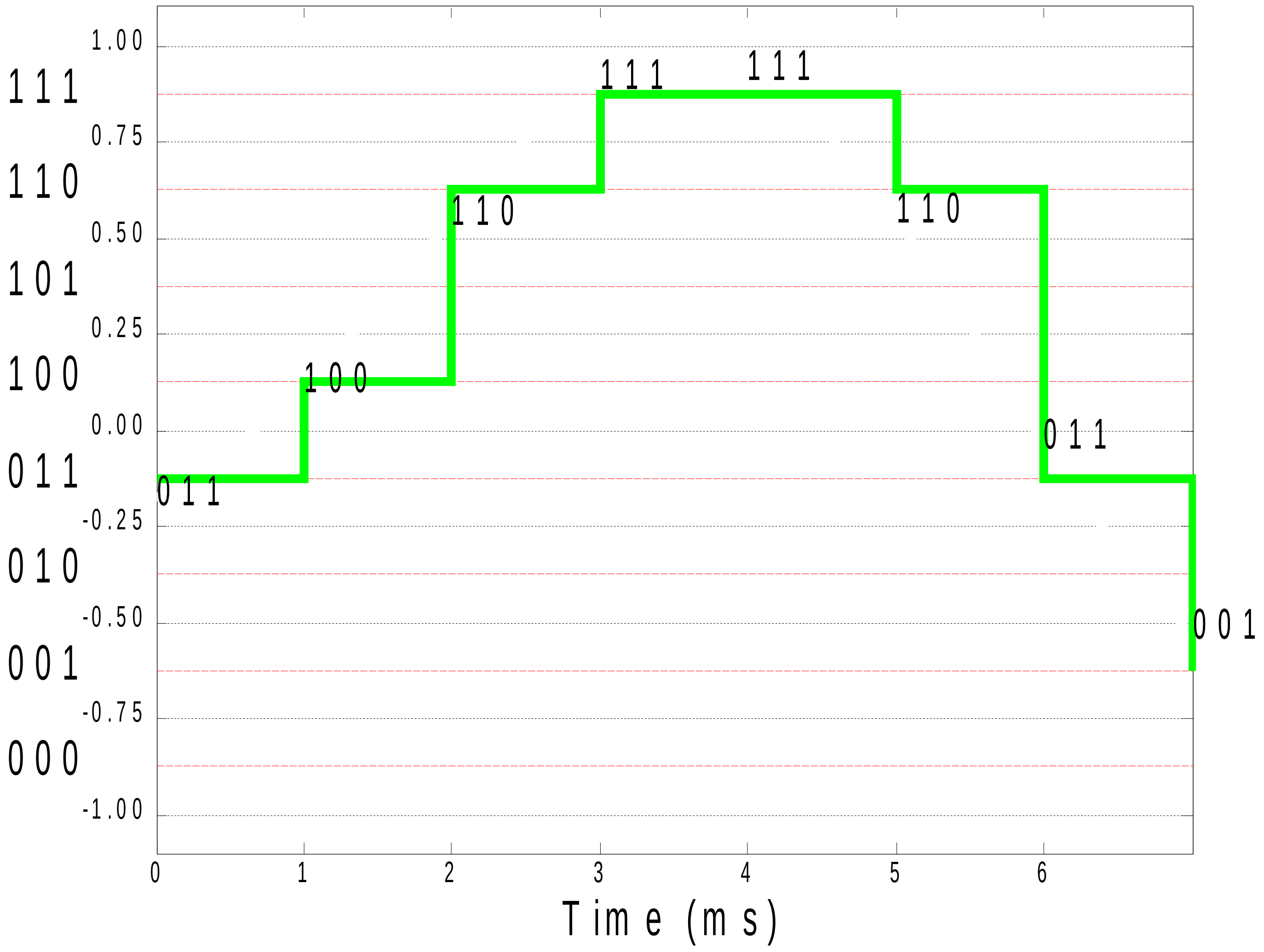


Digital-to-analog (D/A) conversion

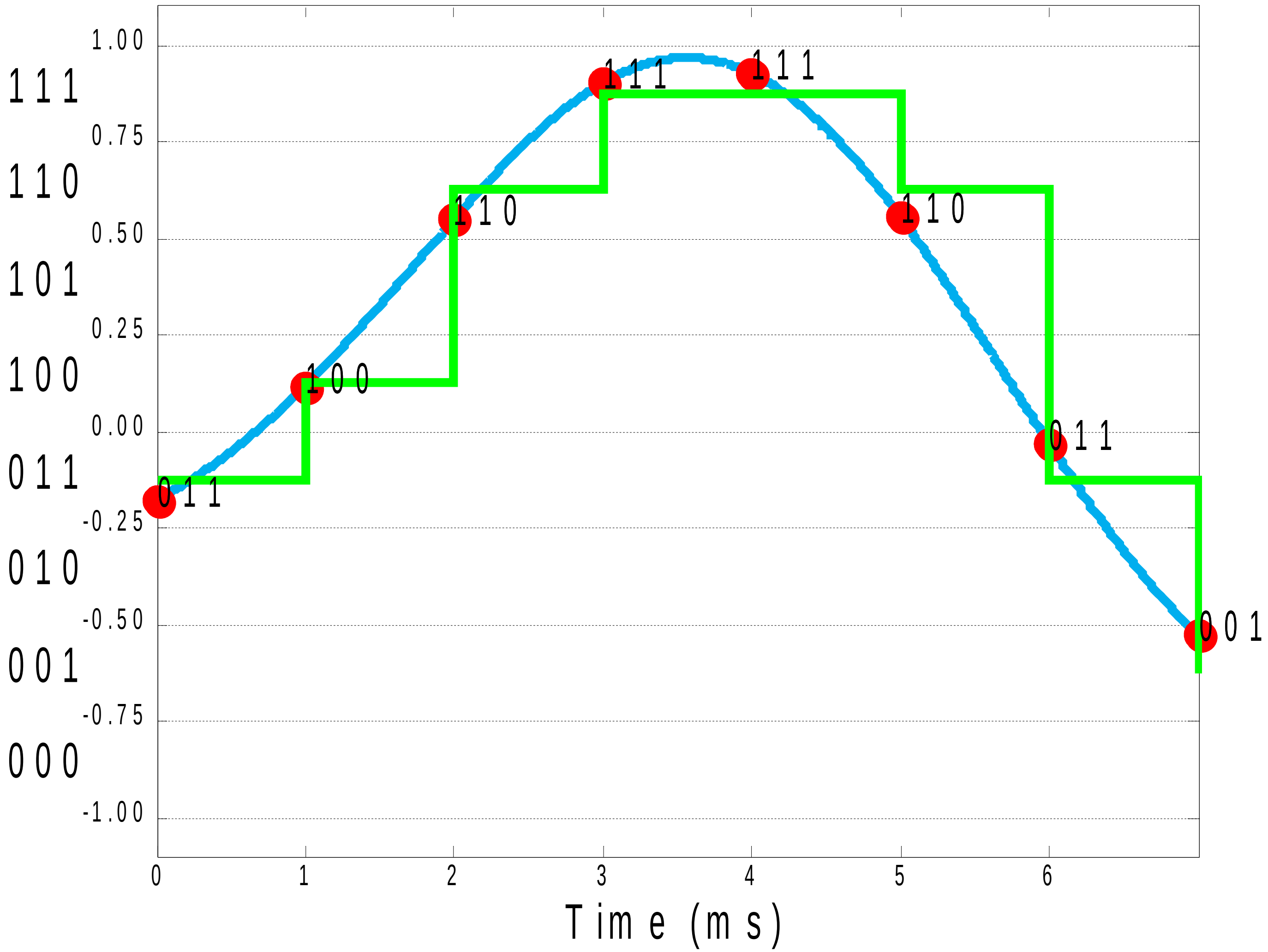
- The reconstruction levels are the midpoints of the intervals used by the quantizer.



Digital-to-analog (D/A) conversion



Digital-to-analog (D/A) conversion



Example Problem 2

Consider the following analog waveform. This waveform is sampled at a 1-kHz rate and quantized with a 4-bit quantizer (input range -1.0 to +1.0 V).

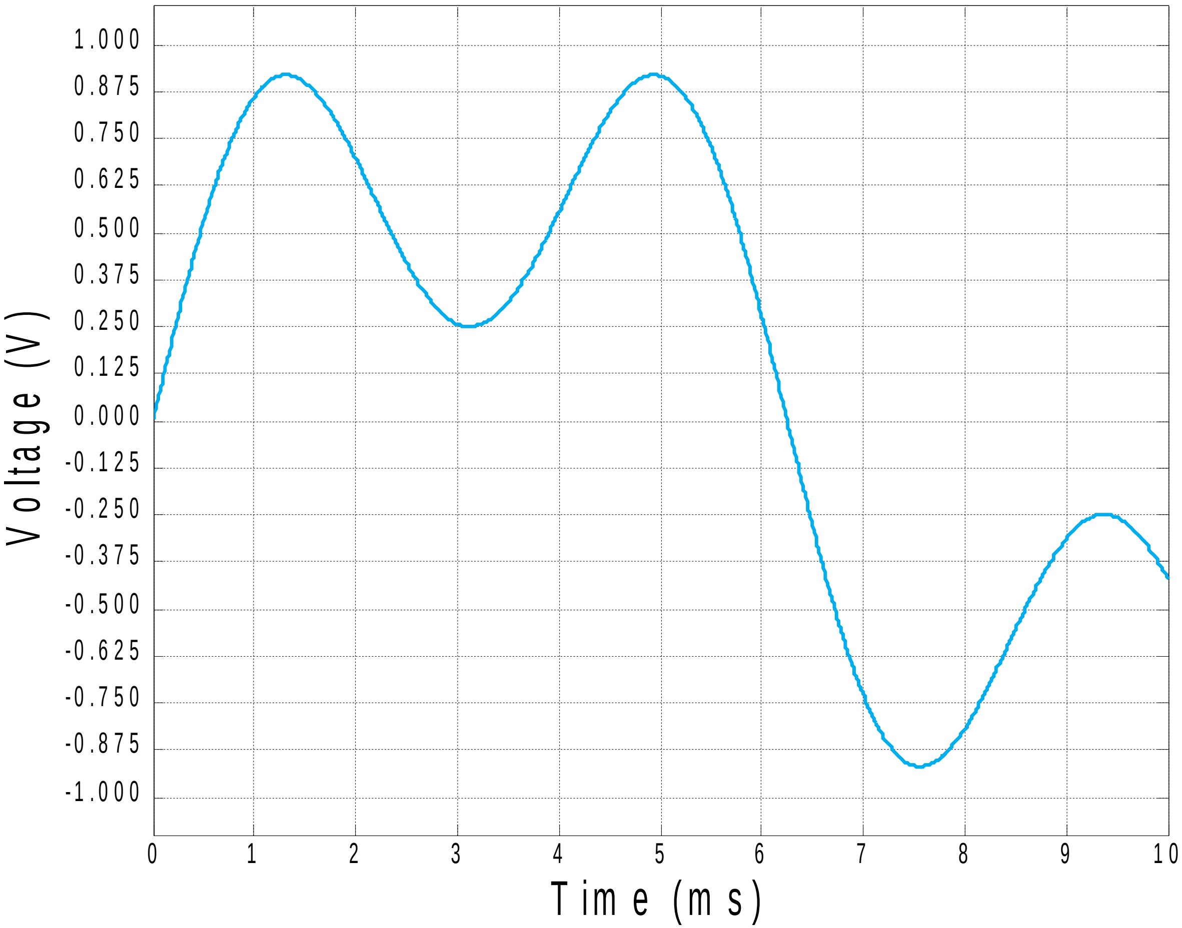
a. Circle the sample points.

b. Indicate the quantization intervals and corresponding binary values.

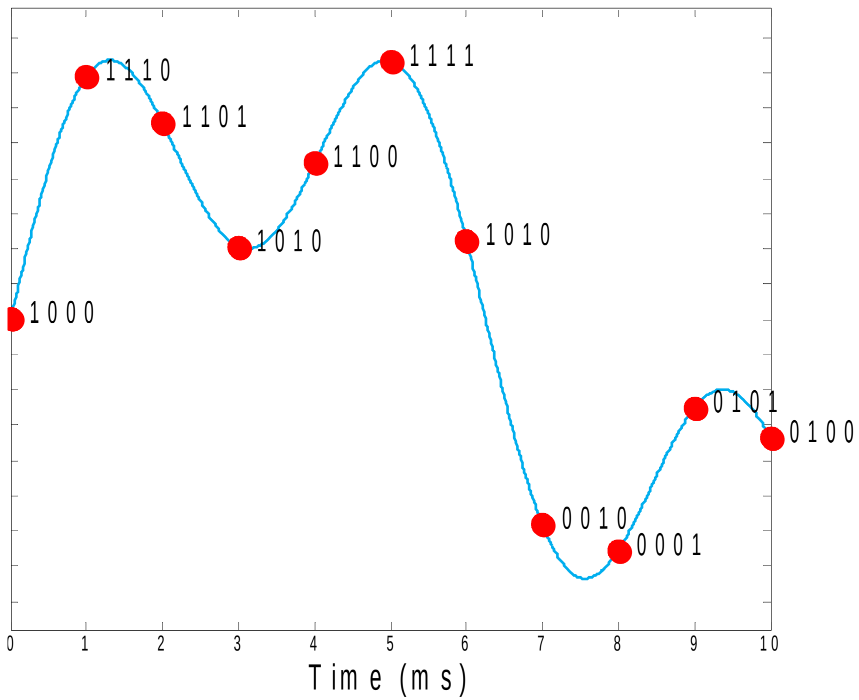
c. Indicate the binary number assigned to each sample point.

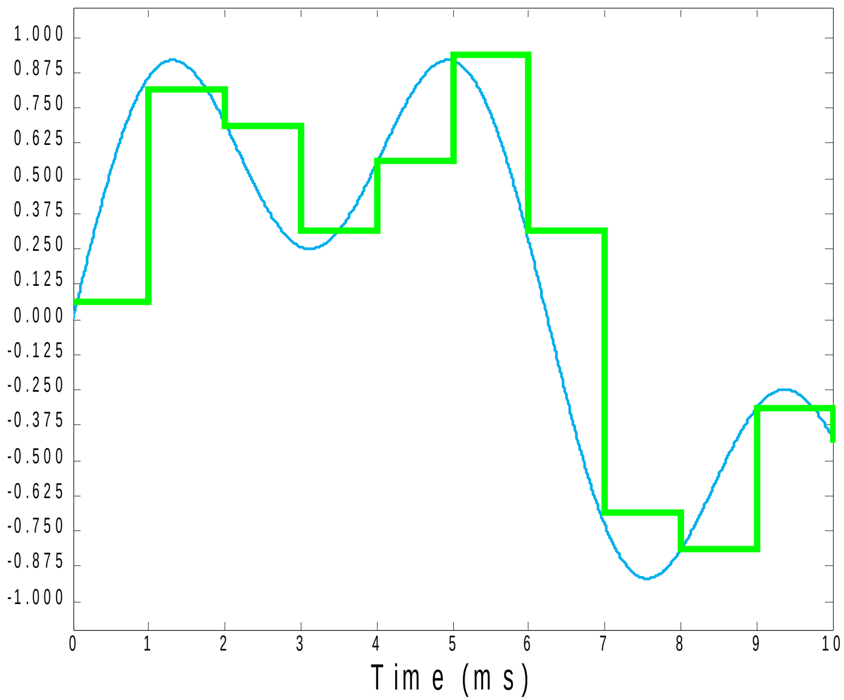
d. Sketch the reconstructed waveform at the D/A.

Example Problem 2



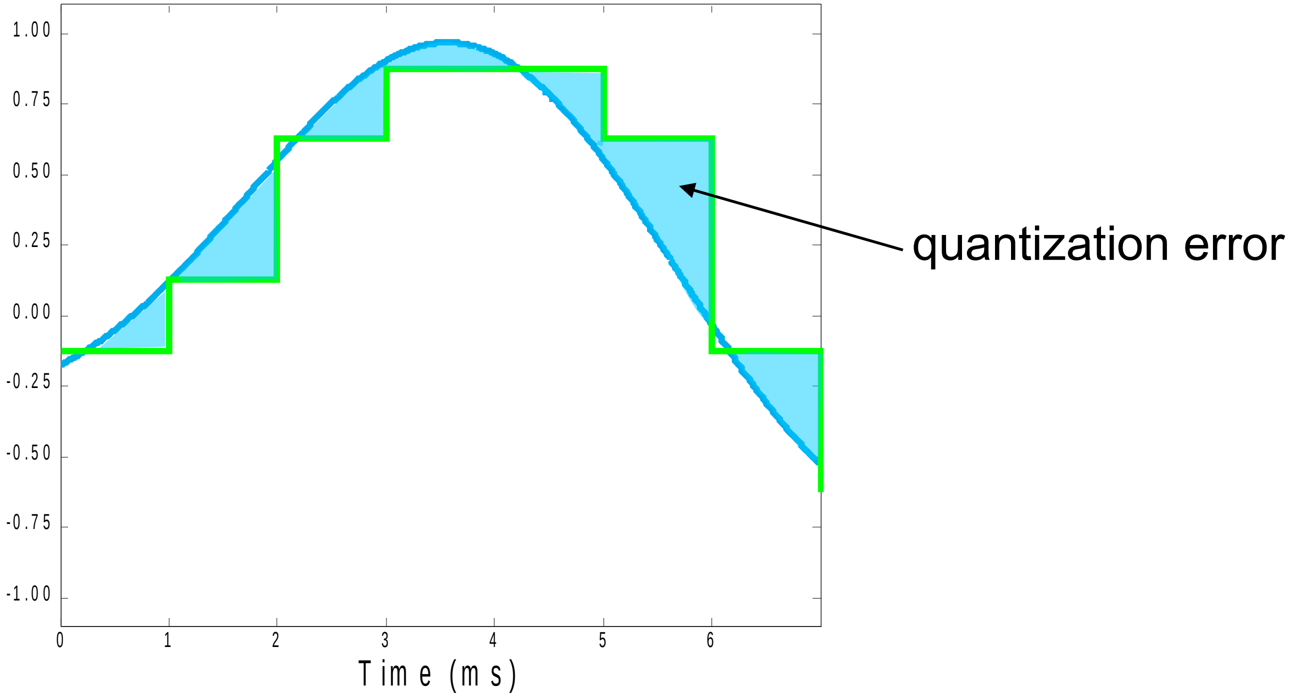
Problem 2 solution





Quantization error

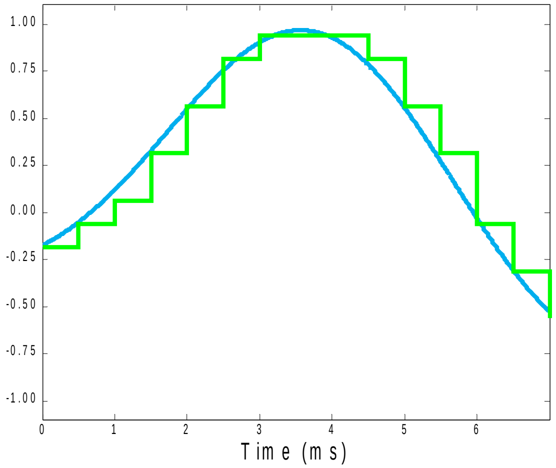
- Notice that there is some error associated with this conversion process.
  - This error is the difference between analog input and the reconstructed signal.



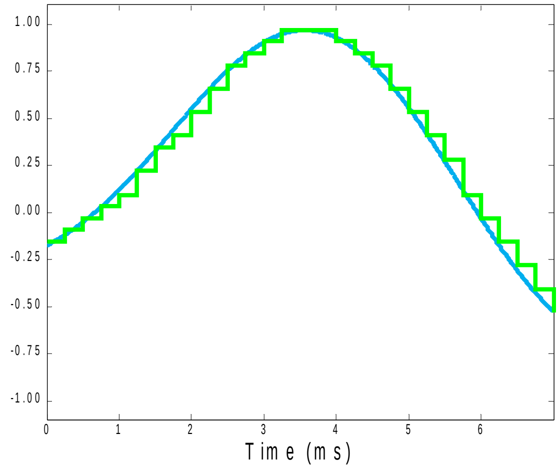
Quantization error

- Quantization error can be reduced by increasing the bits *N* and the sampling rate.

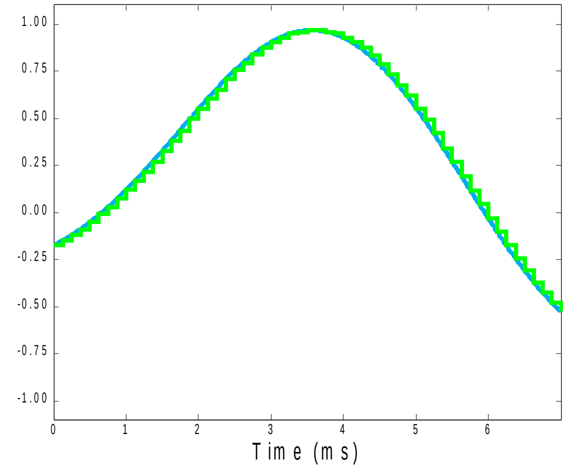
4-bit quantization
sampling frequency $f = 2$ kHz



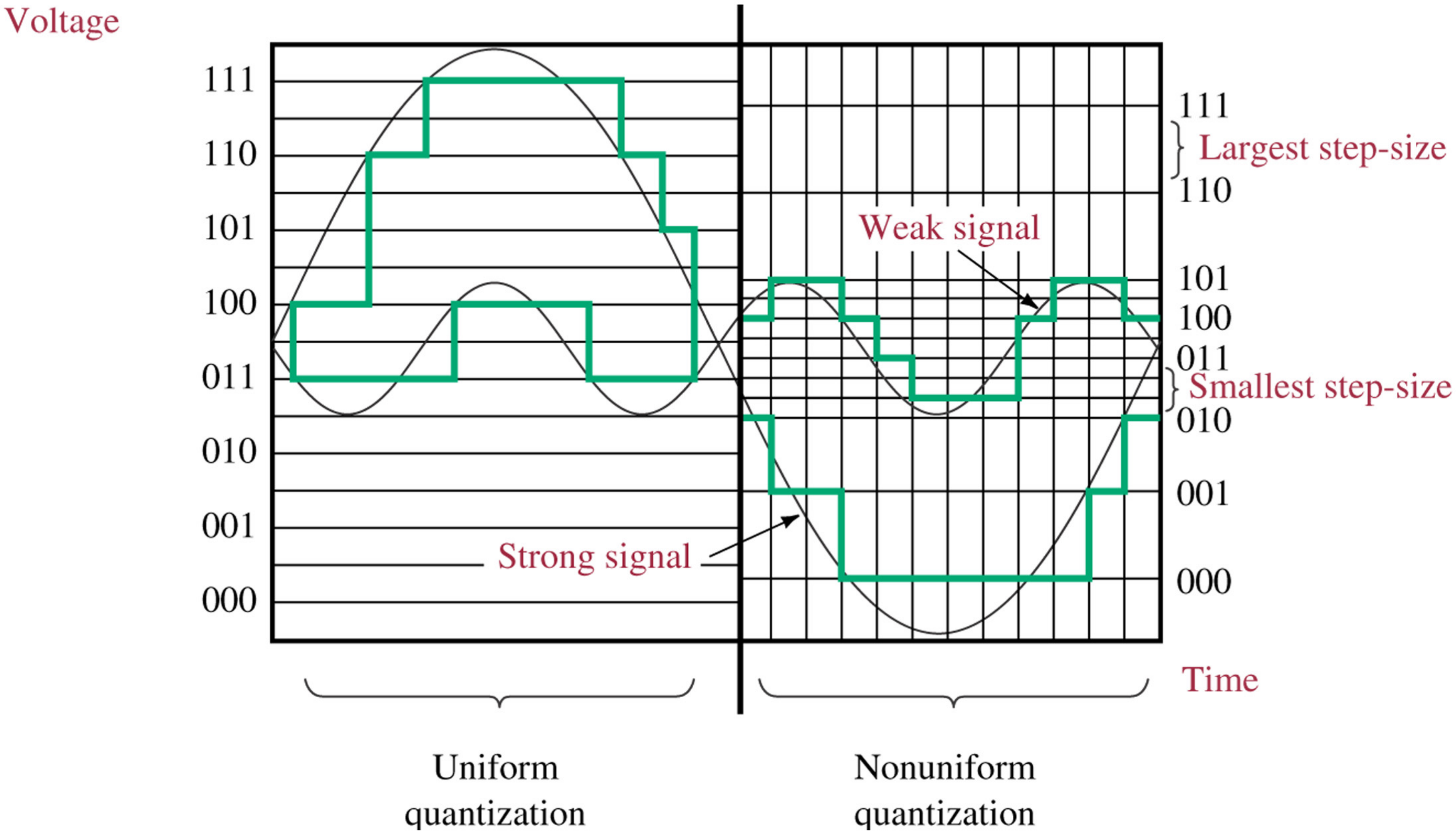
5-bit quantization
sampling frequency $f = 4$ kHz



8-bit quantization
sampling frequency $f = 8$ kHz



Quantization Schemes

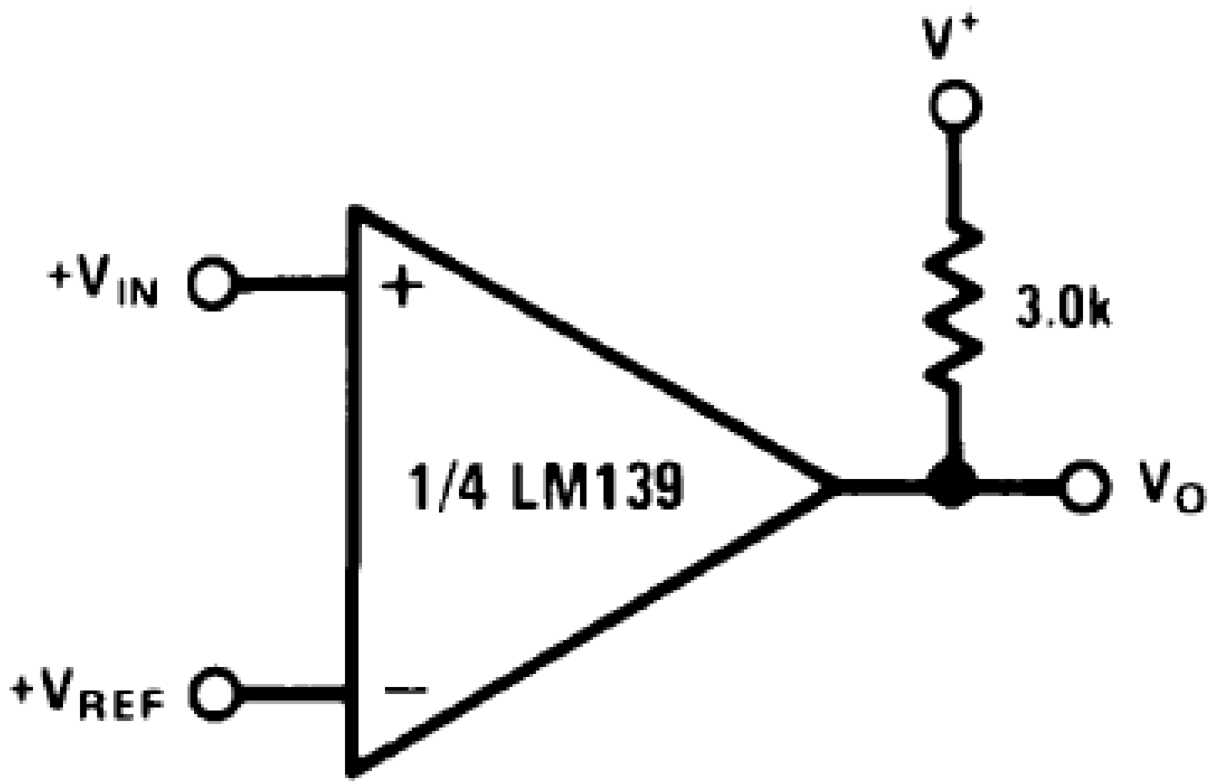


Great!

But HOW do you do it?

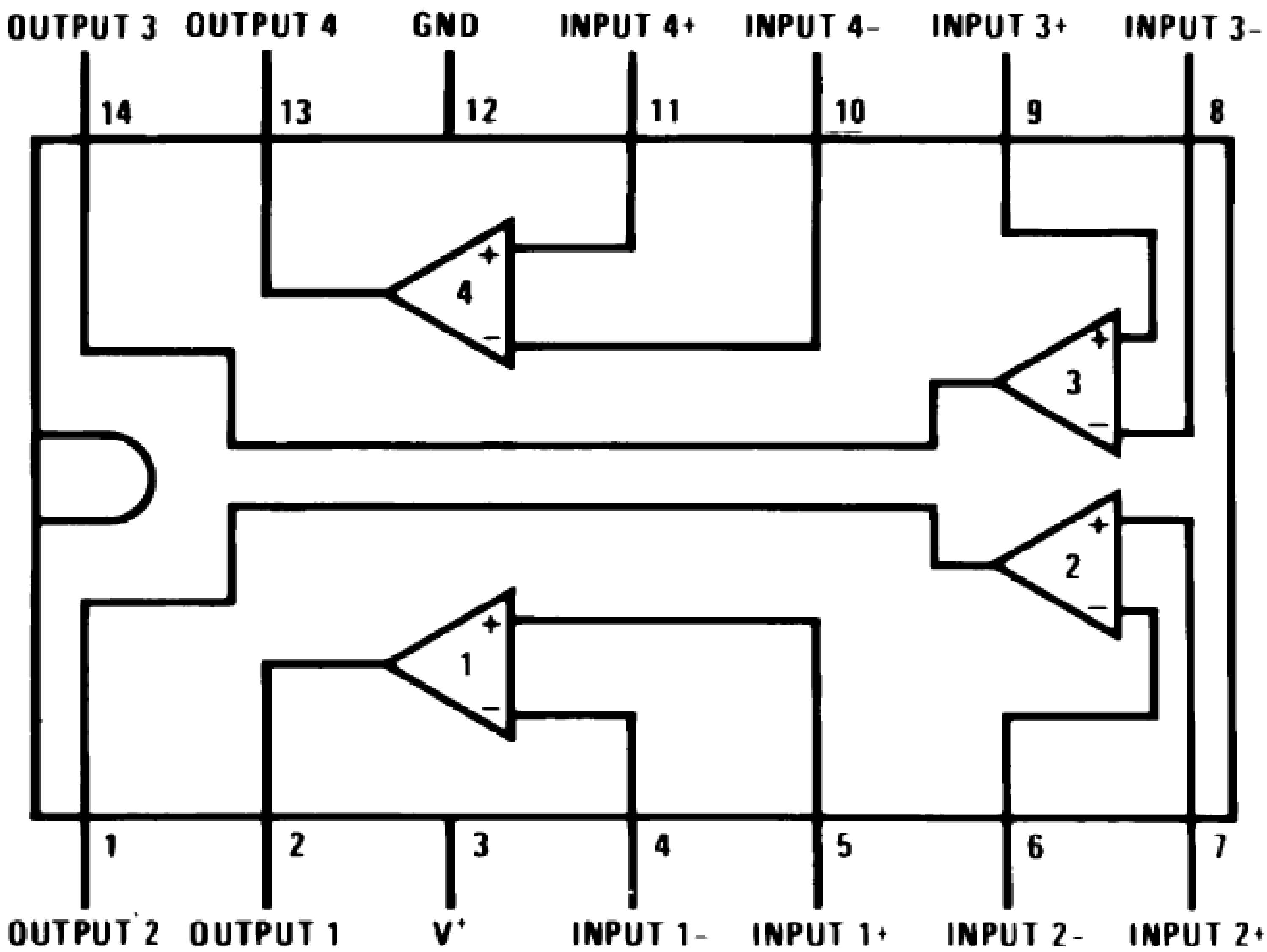
(enough already!)

Basic Comparator

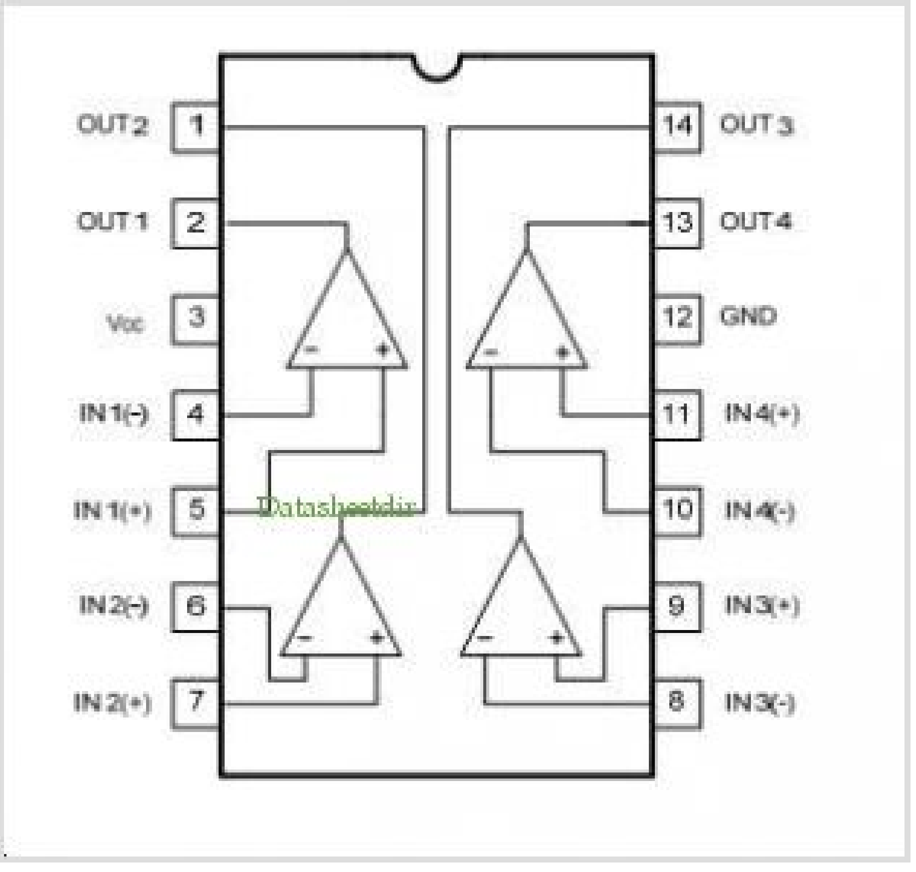



```
if(Vin > Vref)
        Vo = logic 1
else
        Vo = logic 0
```

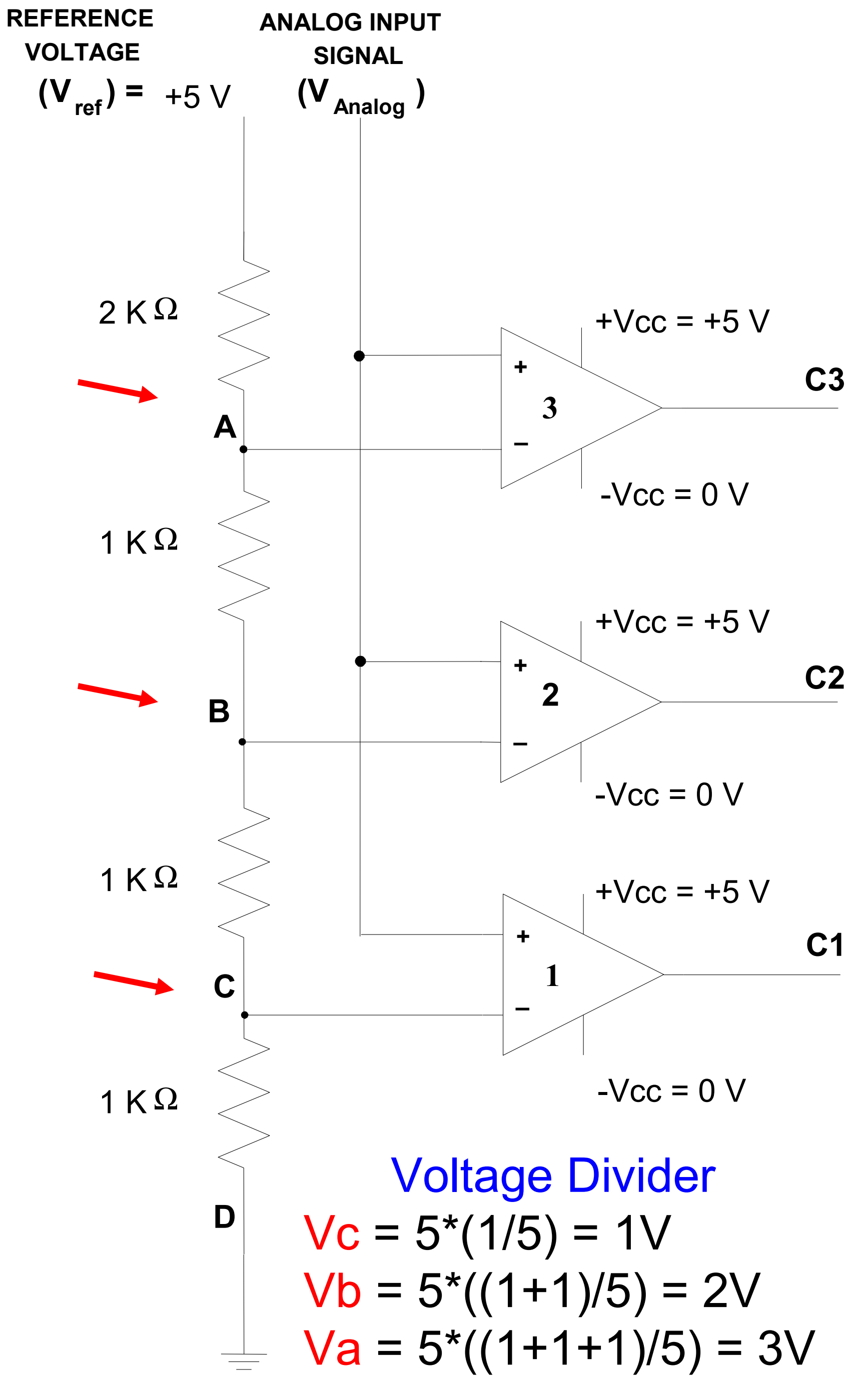
Dual-In-Line Package

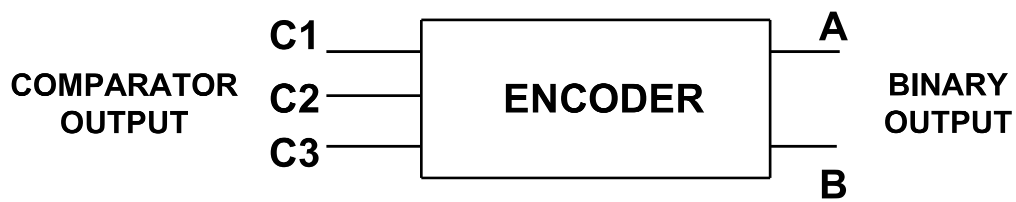



PM206BX
LM339N


OUT2
1
OUT1
2
Vcc
3
IN1(-)
4
IN1(+)
5
IN2(-)
6
IN2(+)
7
14
OUT3
13
OUT4
12
GND
11
IN4(+)
10
IN4(-)
9
IN3(+)
8
IN3(-)
Datasheetdir

“Flash” ADC





Voltage Divider

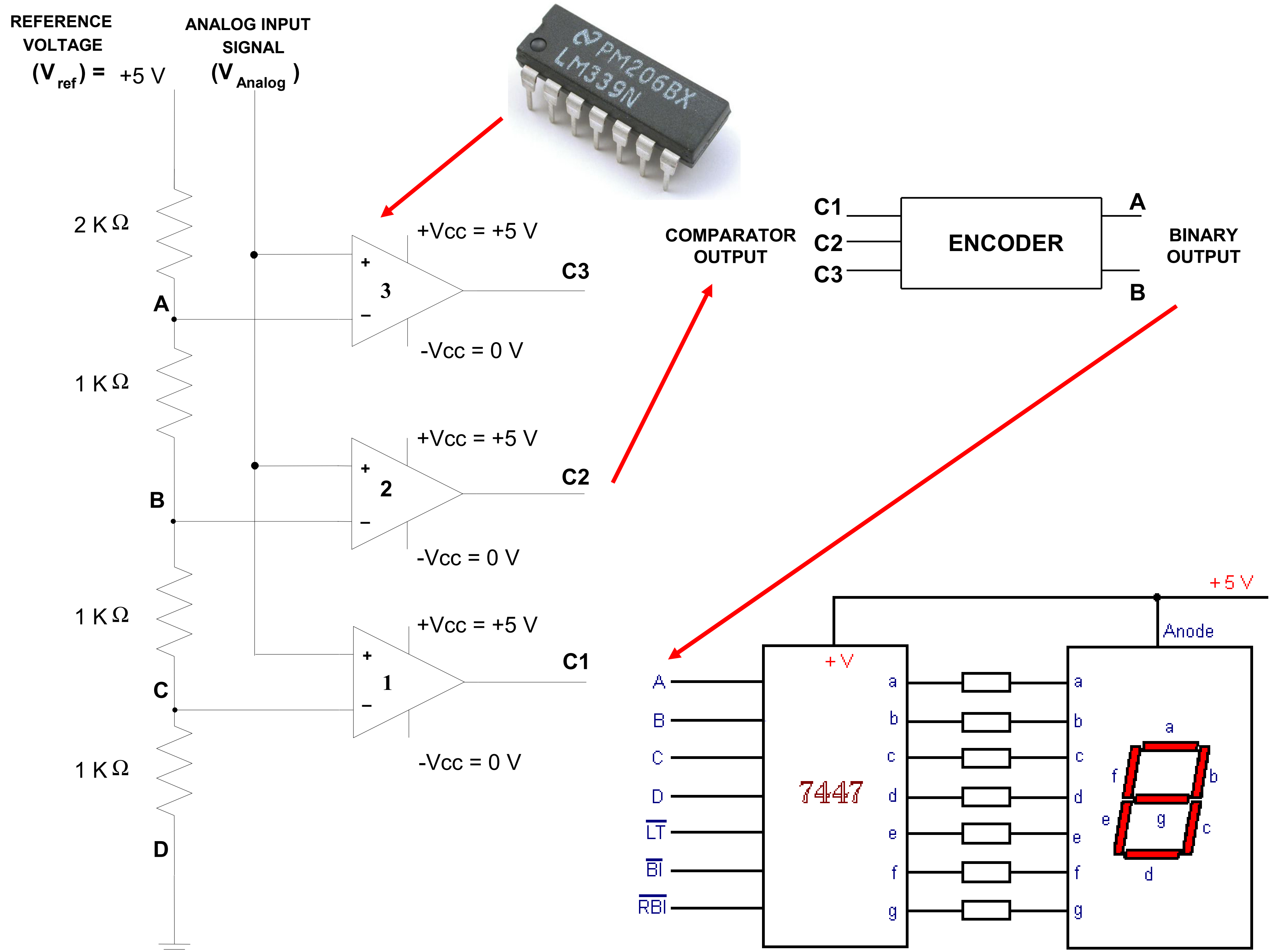
Vc = 5*(1/5) = 1V

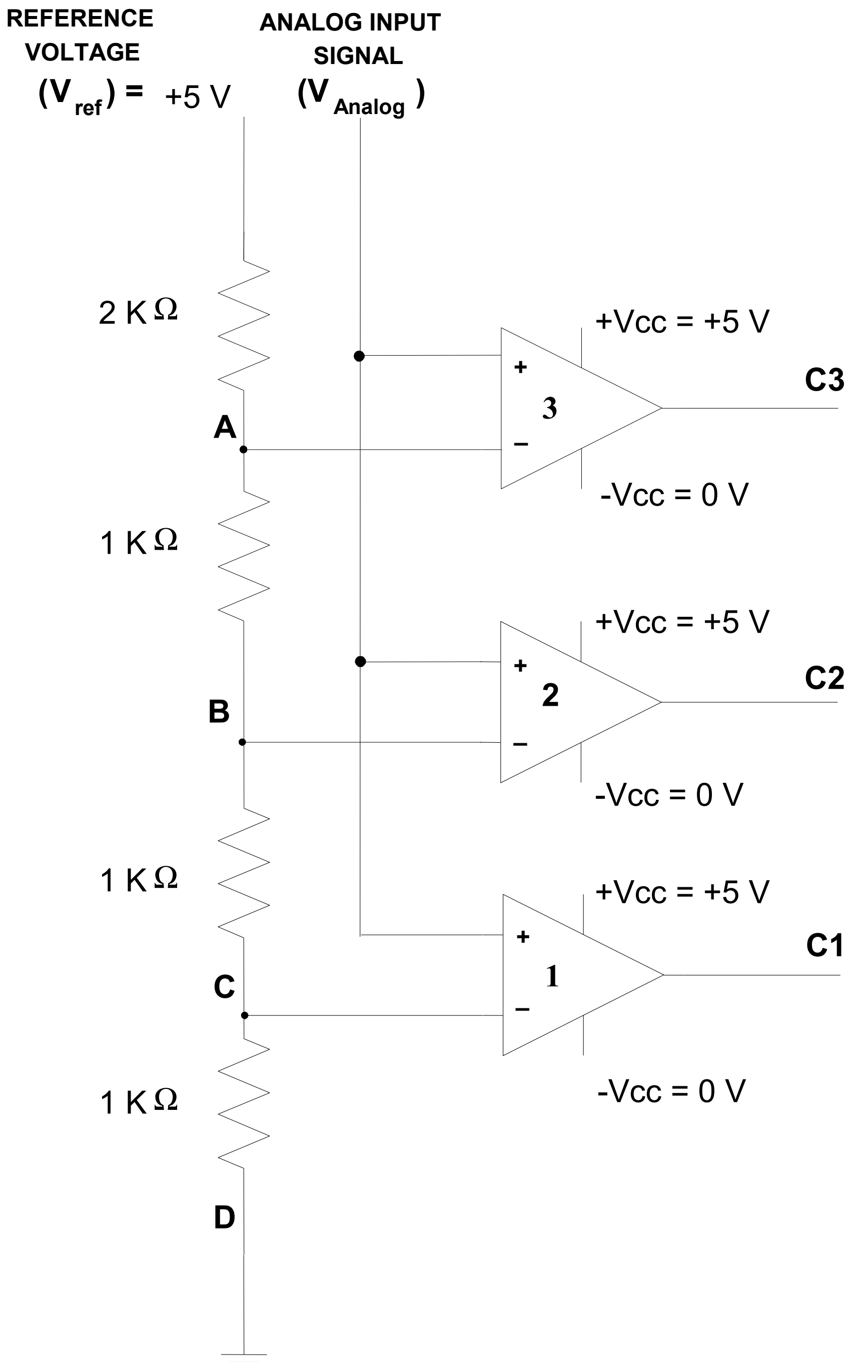
Vb = 5*((1+1)/5) = 2V

Va = 5*((1+1+1)/5) = 3V

C3	C2	C1		A	B		Decimal #
0	0	0		0	0		0
0	0	1		0	1		1
0	1	1		1	0		2
1	1	1		1	1		3

Figure 4. Encoder Output

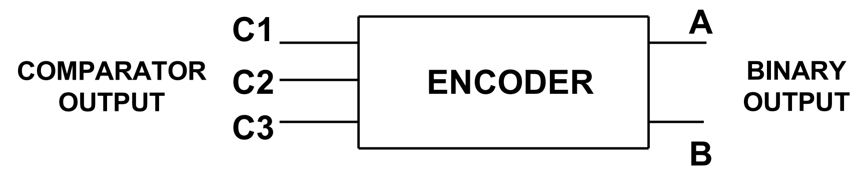

REFERENCE VOLTAGE
(Vref) = +5 V
ANALOG INPUT SIGNAL
(V Analog)
2 K Ω
1 K Ω
1 K Ω
1 K Ω
A
B
C
D
PM206BX
LM339N
+Vcc = +5 V
-Vcc = 0 V
+Vcc = +5 V
-Vcc = 0 V
+Vcc = +5 V
-Vcc = 0 V
3
2
1
C3
C2
C1
COMPARATOR OUTPUT
C1
C2
C3
ENCODER
A
B
BINARY OUTPUT
+5 V
Anode
+V
7447
A
B
C
D
LT
BI
RBI
a
b
c
d
e
f
g
a
b
c
d
e
f
g



Characteristics:

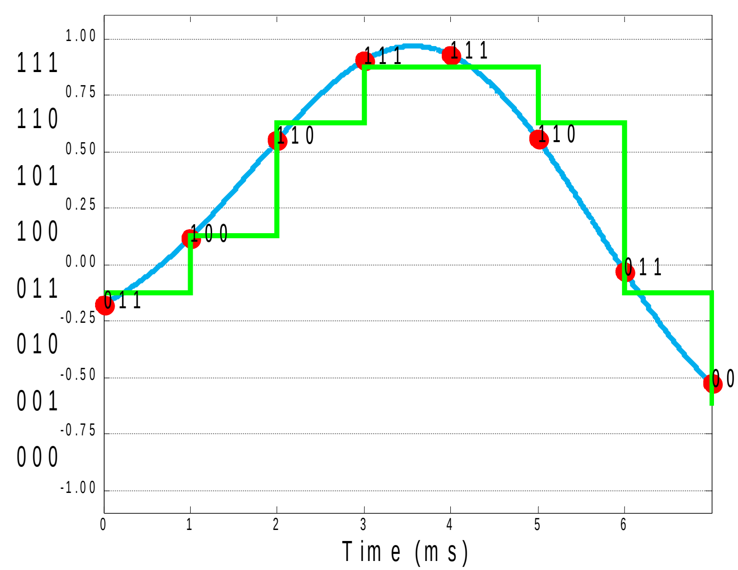
* Fast

* Expensive

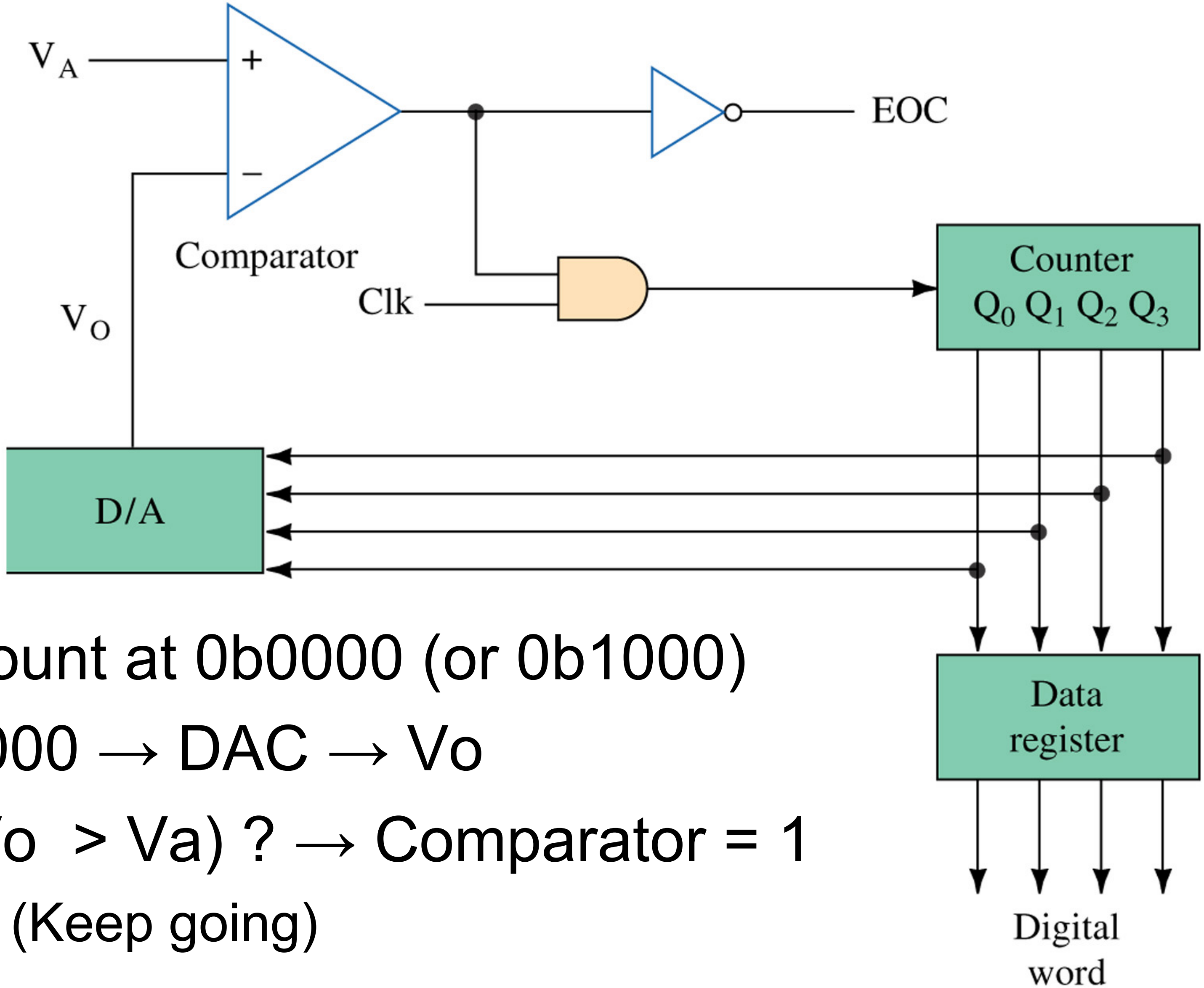


(2^n – 1) comparators for a n bit conversion

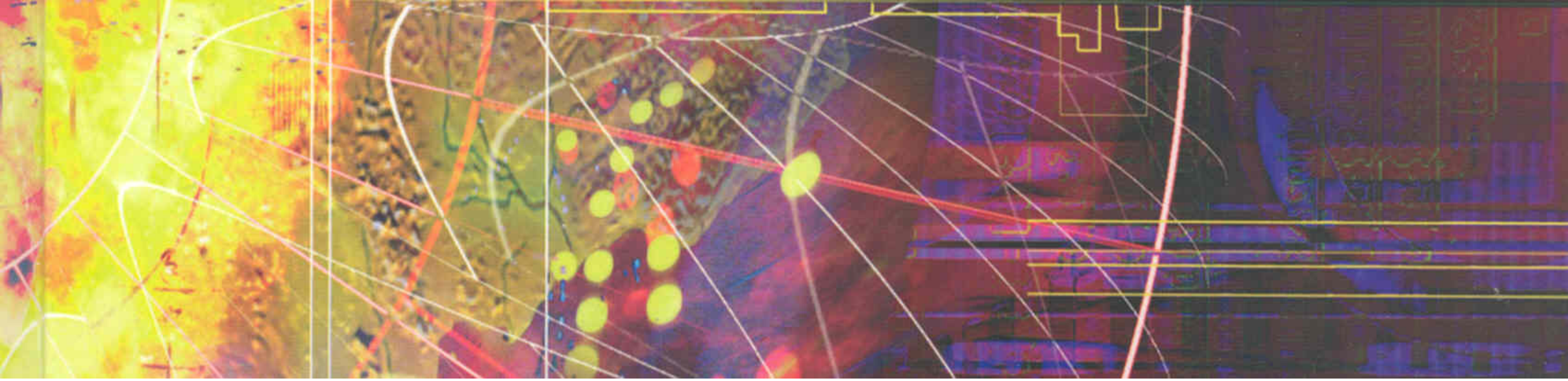
16-bit ADC → 65,535 comparators!



Successive Approximation ADC (Ramp)



- Count at 0b0000 (or 0b1000)
- 0000 → DAC → Vo
- (Vo > Va) ? → Comparator = 1
  - (Keep going)



Thank you!
Questions?

Numbers & Bytes Meeting
04 February 2011 : 4:00 – 5:00 PM
CSCC Room 203

William M. Jones, Ph.D.

More stuff after this.

Lasciate ogne speranza, voi ch'intrate

All hope abandon ye who enter here.

Figure 8-21 DAC input/output.

What is LSB? MSB?
Give example on board

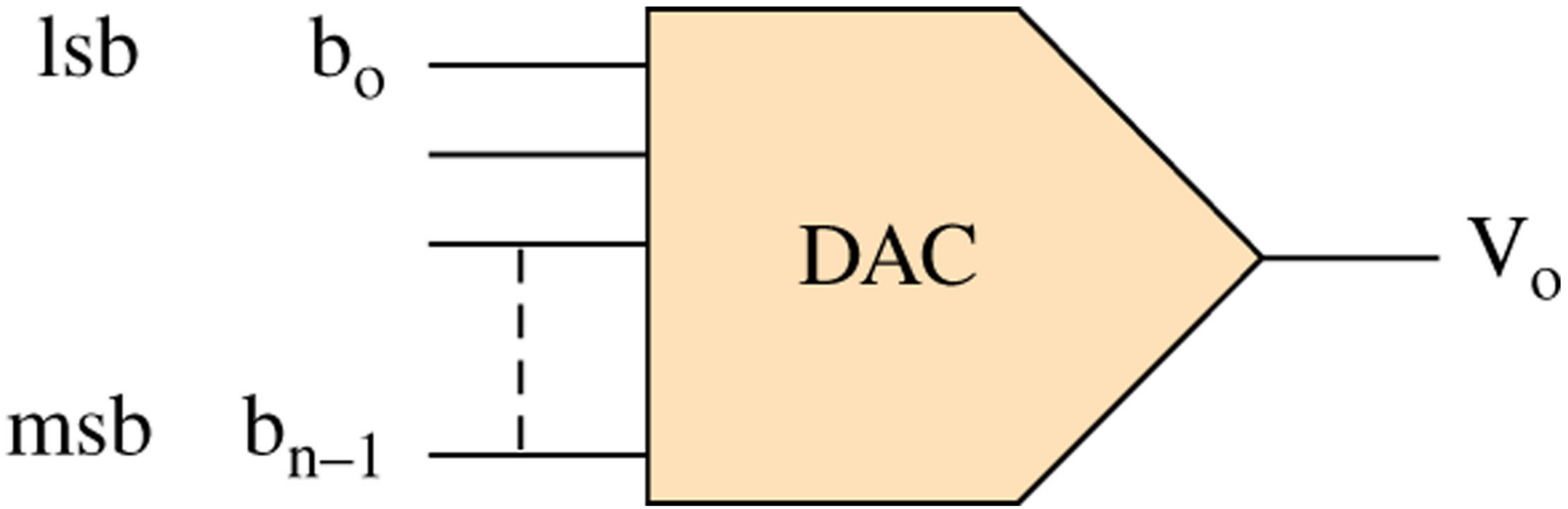


Figure 8-22 Binary-weighted resistor DAC.

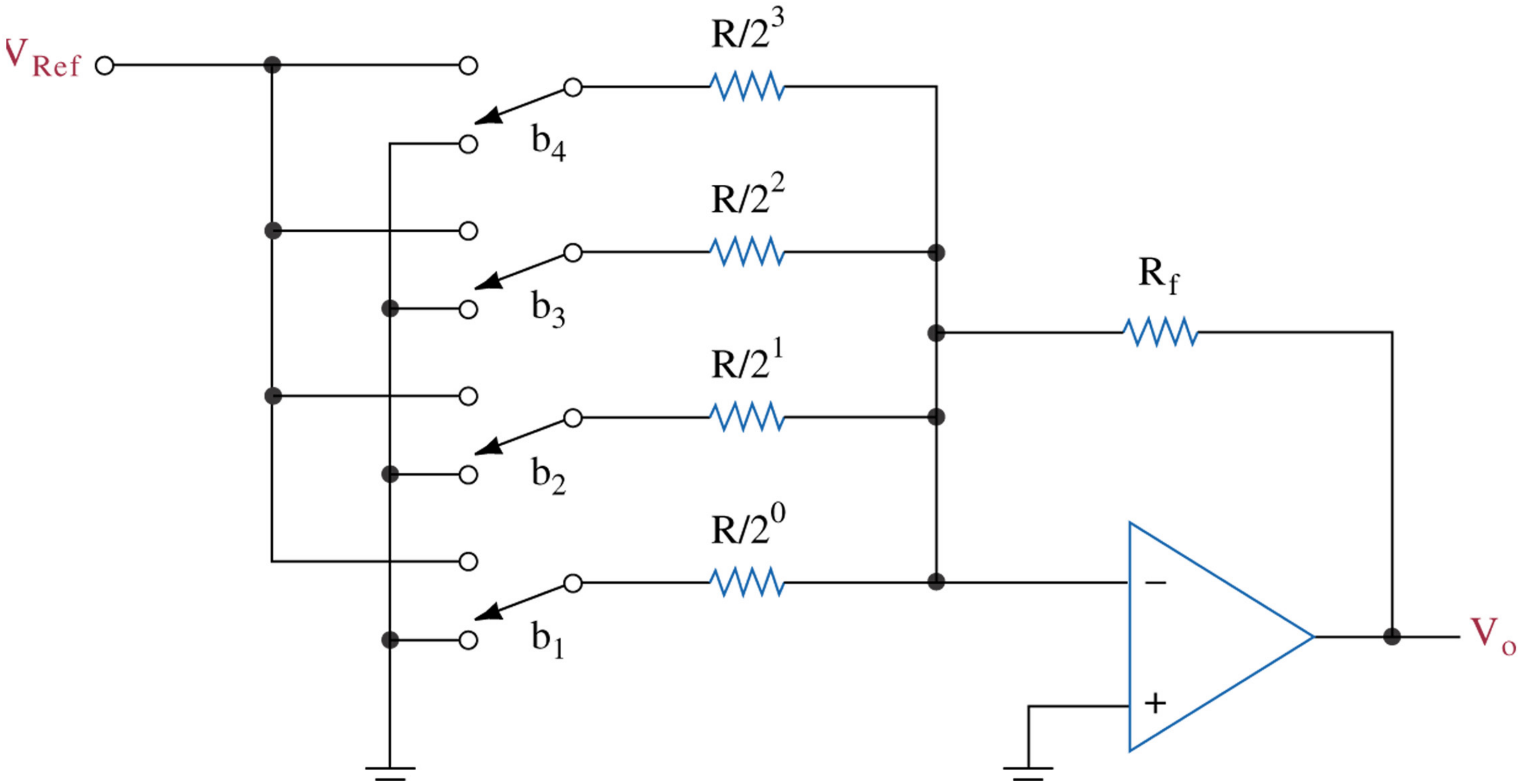
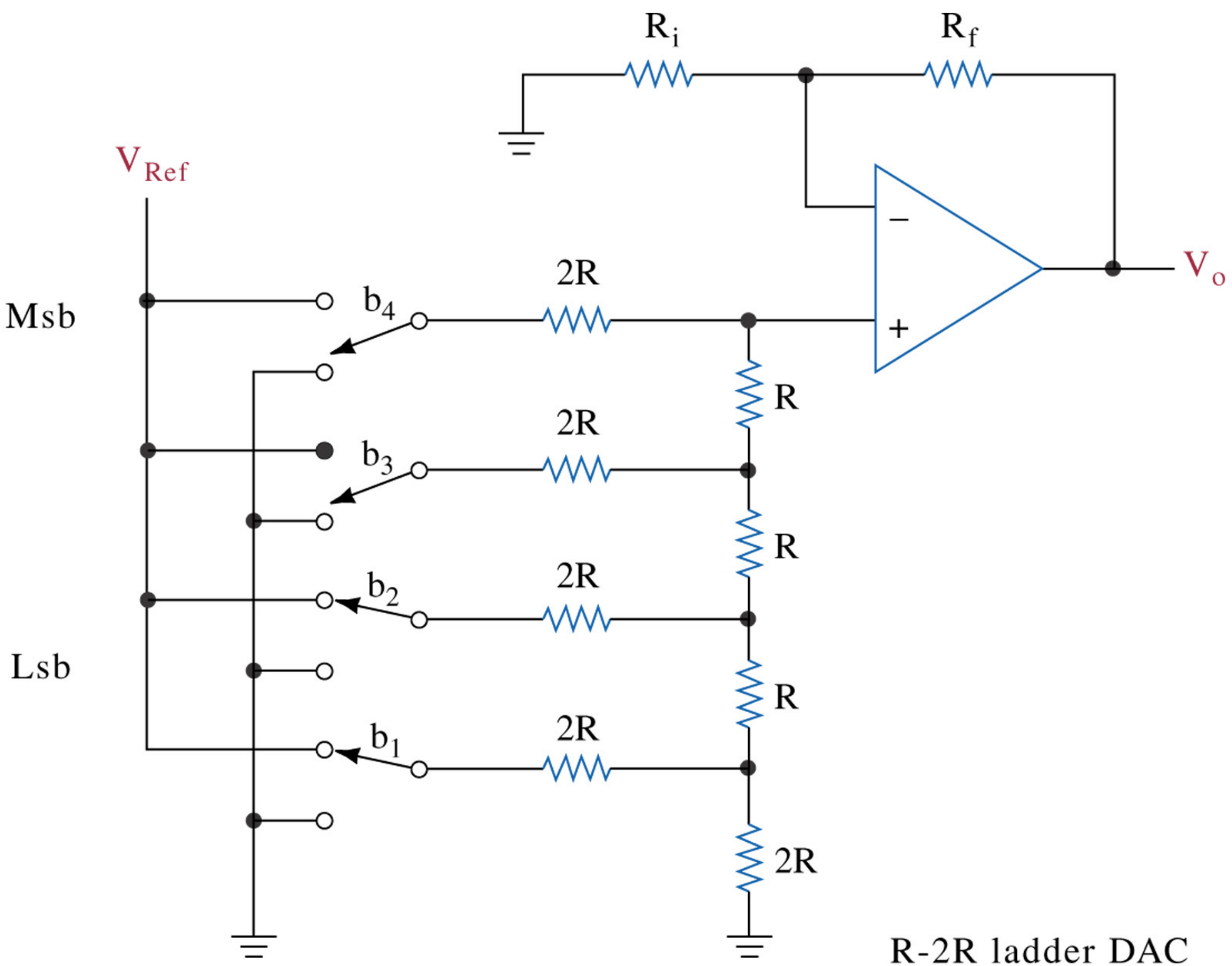
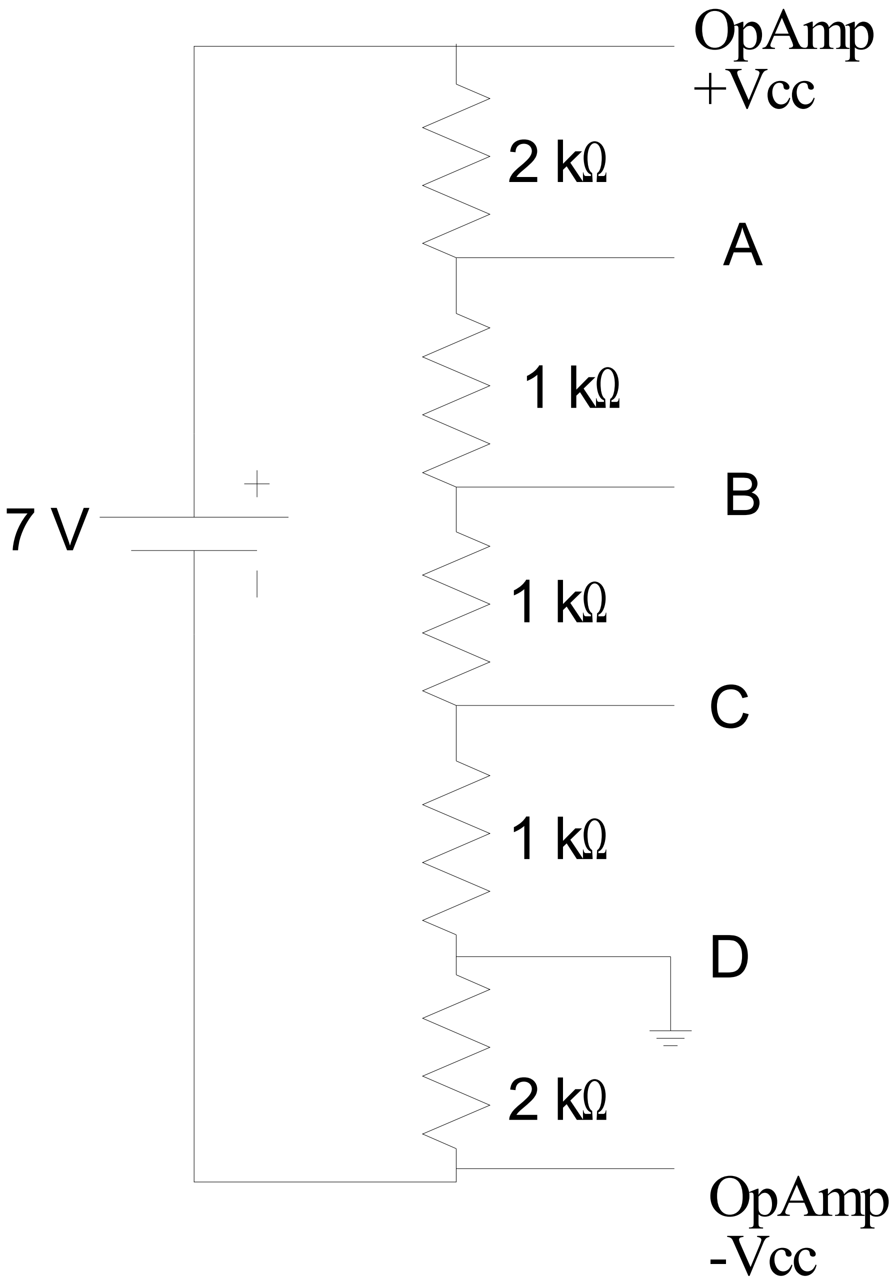


Figure 8-23 R-2R ladder DAC.





How much current flows?

V = IR → I = V/R

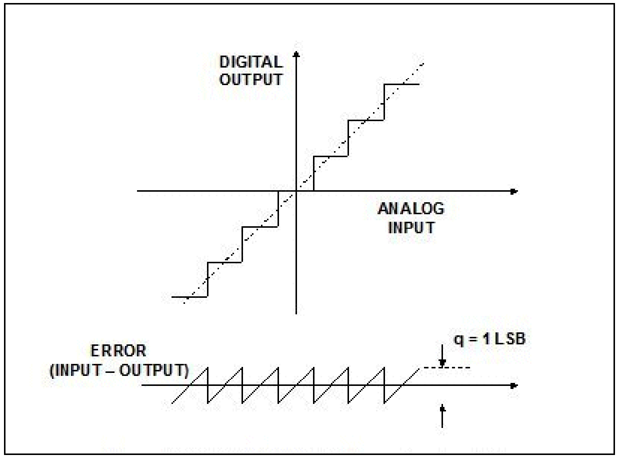
I = 7 / (2+1+1+1+2) = 7/7 = 1

So, how much across each Resistor?

So mow much between A/B/C And ground?

Quantization error

- This error manifests itself as additive noise due to the difference between the analog value and its closest digital value.



- Quantization noise has an approximate rms voltage given

$$V_n = \frac{q}{\sqrt{12}}$$

Dynamic range

- The dynamic range of an A/D converter is the ratio of the maximum input voltage to the minimum recognizable voltage level (q).
- Dynamic range is typically express in decibels and for an N-bit quantizer is given

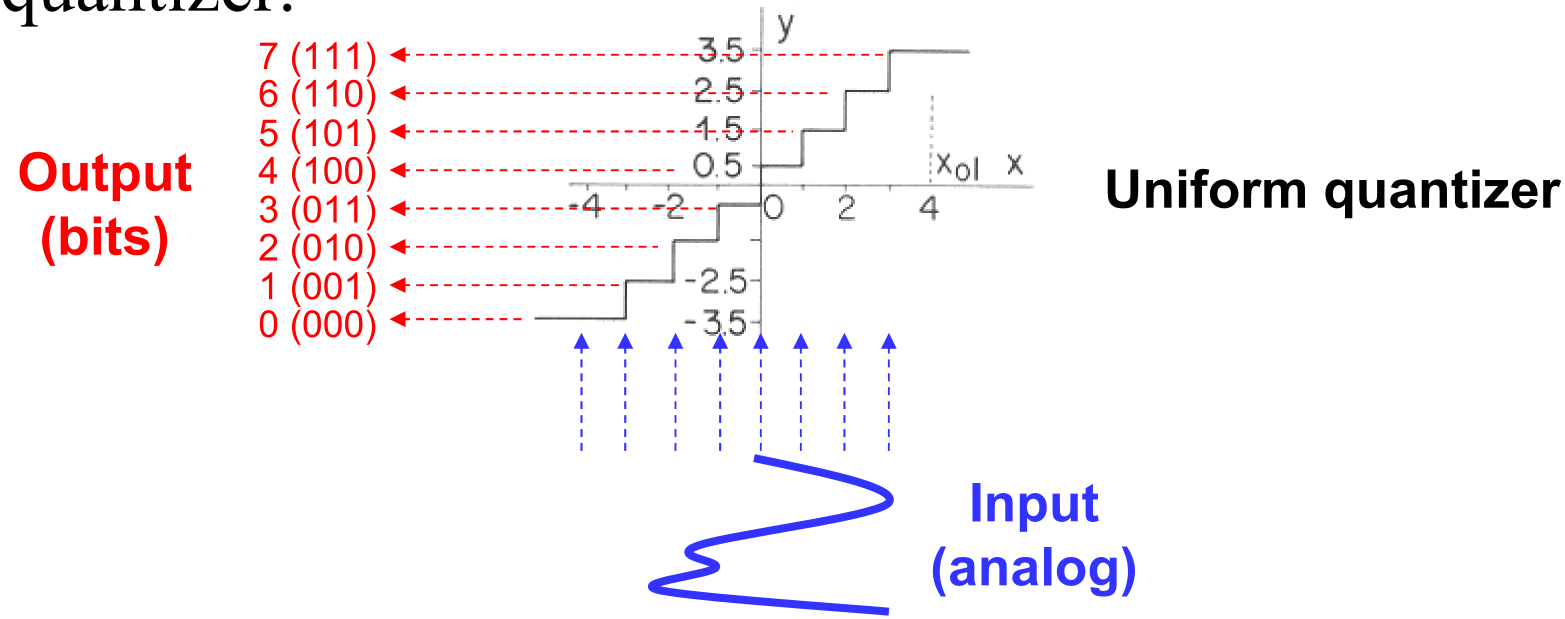
$$\mathrm{DR} = 20\log\frac{v_{\max} - v_{\min}}{q} = 20\log 2^{N} = 6.02\cdot N \quad [\mathrm{dB}]$$

Example Problem 3

What is the dynamic range of an 8-bit quantizer used for digitizing telephone signals?

Uniform quantization

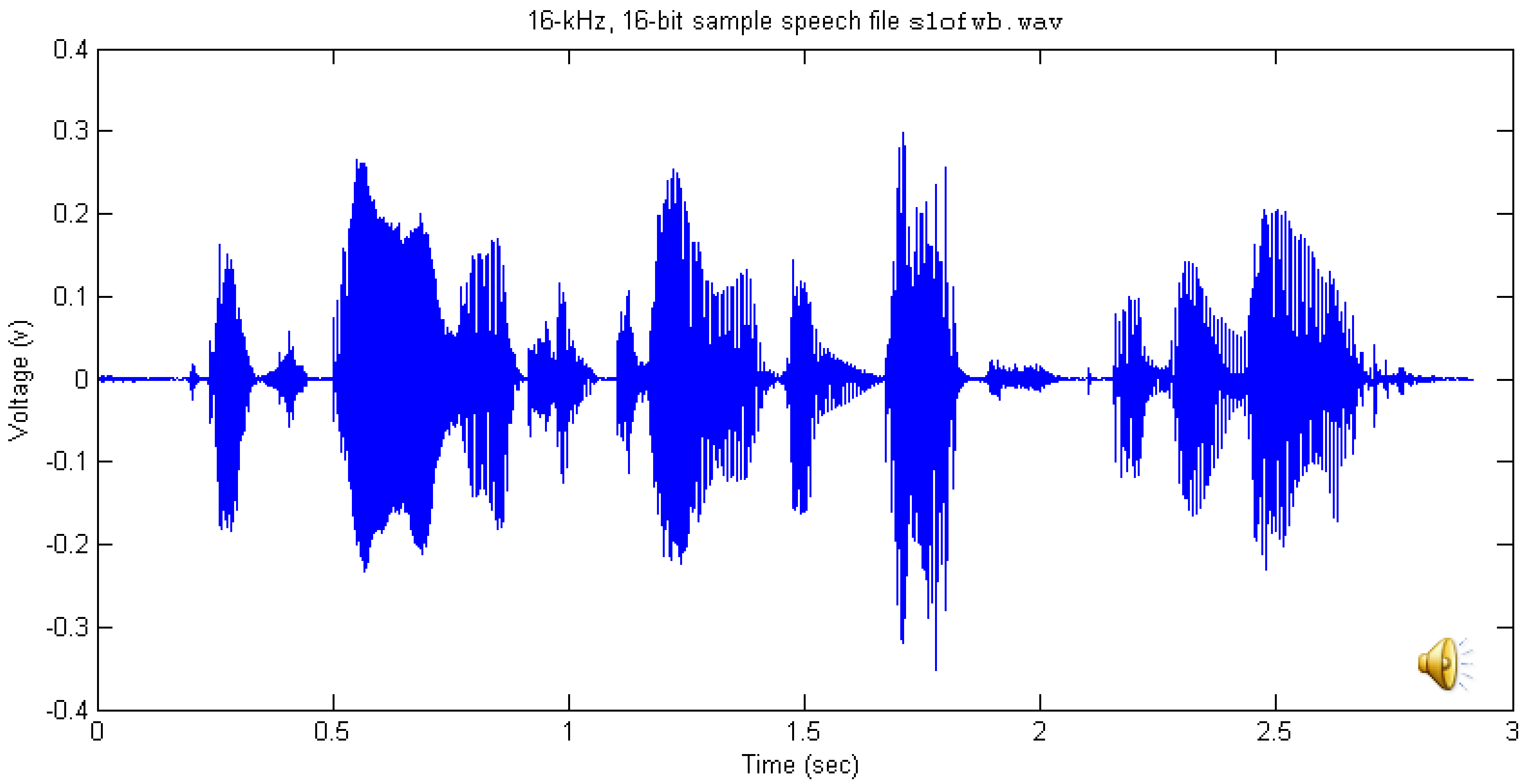
- Thus far we have assumed equal spacing between all of our quantizer levels. This is called a **uniform quantizer**.
- This is a good choice for signals whose values are uniformly distributed across the range of the quantizer.



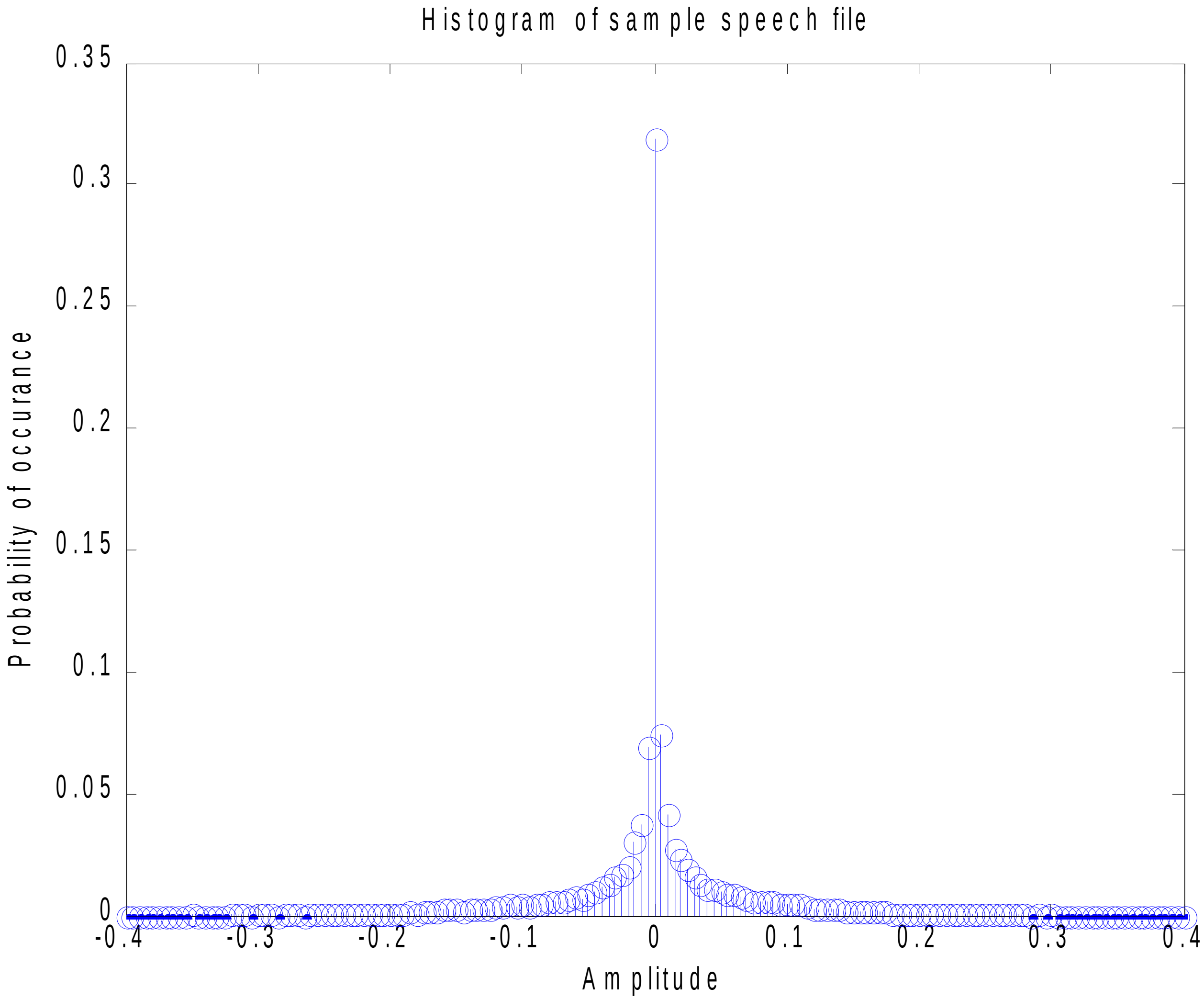
Non-uniform quantization

- Many real-life signals are *not* uniformly distributed.
- In speech signals, small amplitudes occur more frequently than large amplitudes.

Non-uniform quantization



Non-uniform quantization

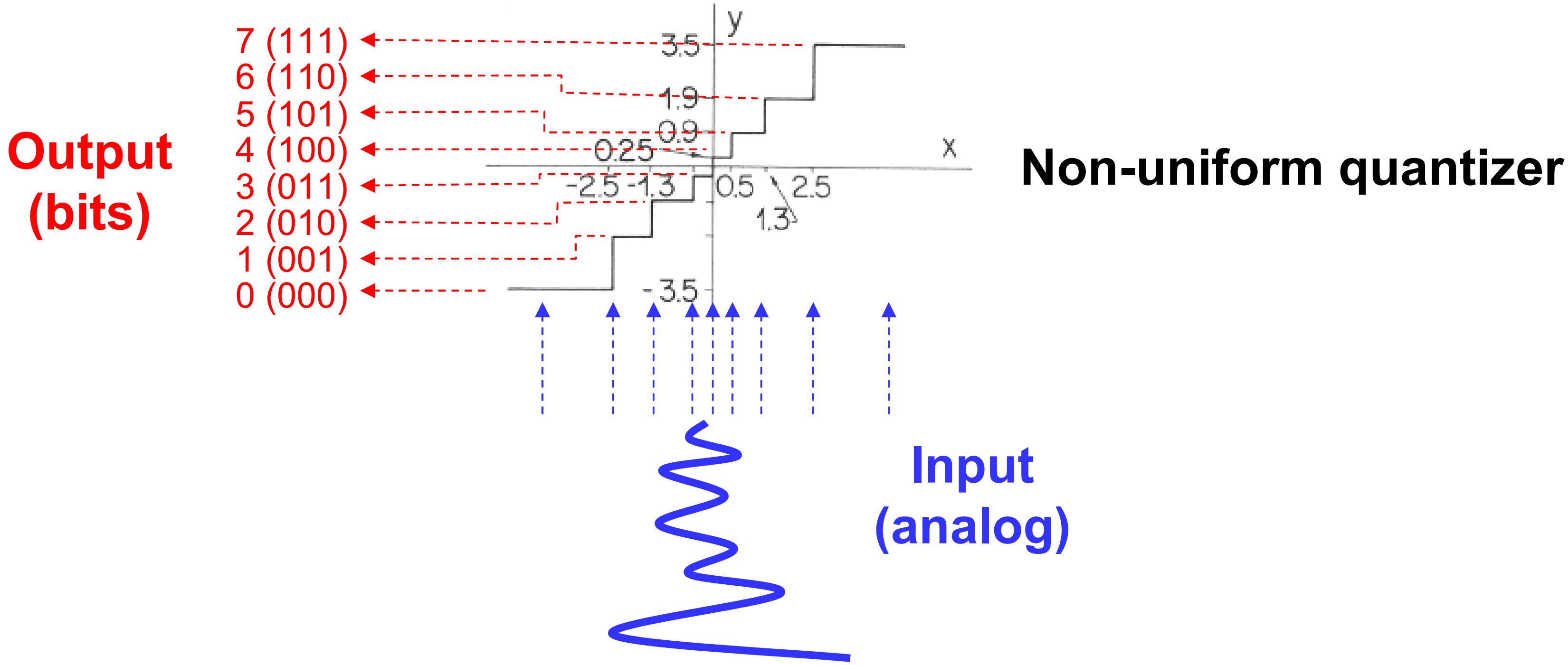


Non-uniform quantization

- Because small amplitudes occur more frequently, it makes sense to improve the resolution of the quantizer at small amplitudes.
- A **non-uniform quantizer** accomplishes this by having quantization levels in are not a fixed size.

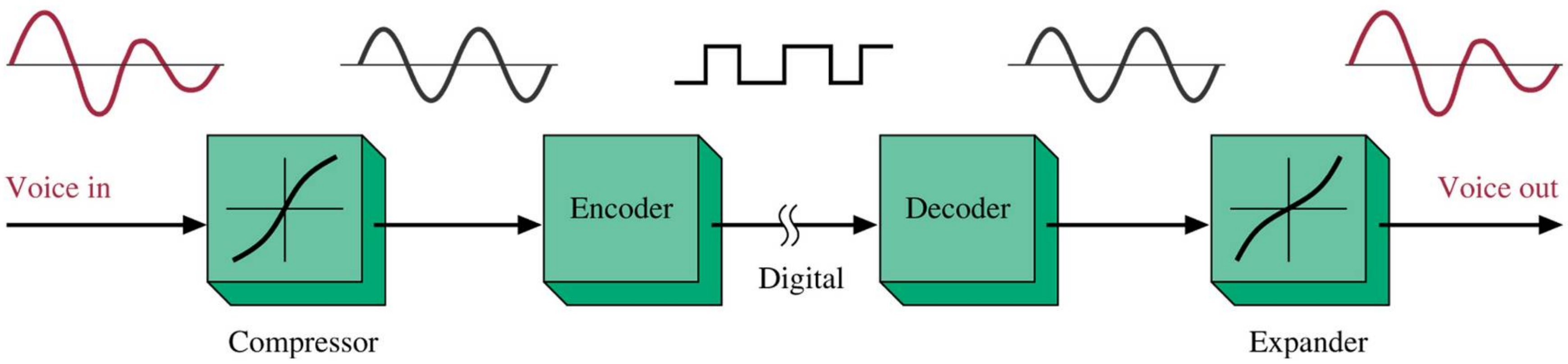
Non-uniform quantization

- A **non-uniform quantizer** accomplishes this by having quantization levels in are not a fixed size.
- This will result in reduced quantization error which improves *S/N*.

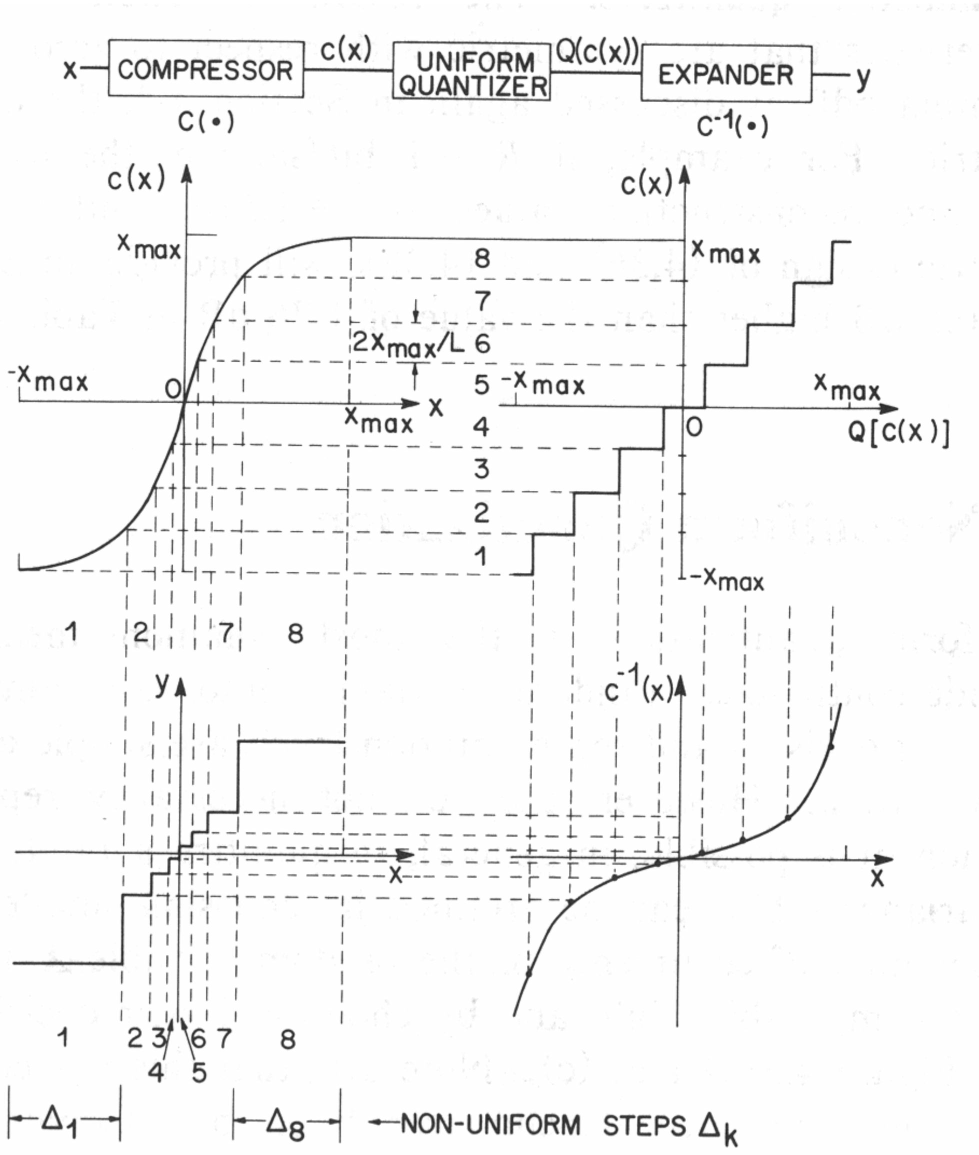


Companding

- One way of realizing **non-uniform quantization** with a uniform quantizer is through a process called companding.
- **Companding** (*compressing* and *expanding*) involves compressing a signal, quantizing it, and then expanding it when it is converted back to analog.



Companding


x
COMPRESSOR
C(•)
c(x)
UNIFORM
QUANTIZER
Q(c(x))
EXPANDER
C-1(•)
y
c(x)
xmax
-xmax
0
xmax
x
2xmax/L
8
7
6
5
4
3
2
1
c(x)
xmax
-xmax
xmax
0
Q[c(x)]
-xmax
1
2
7
8
y
x
c-1(x)
x
1
2 3
4
5
6 7
8
Δ1
Δ8
NON-UNIFORM STEPS Δk

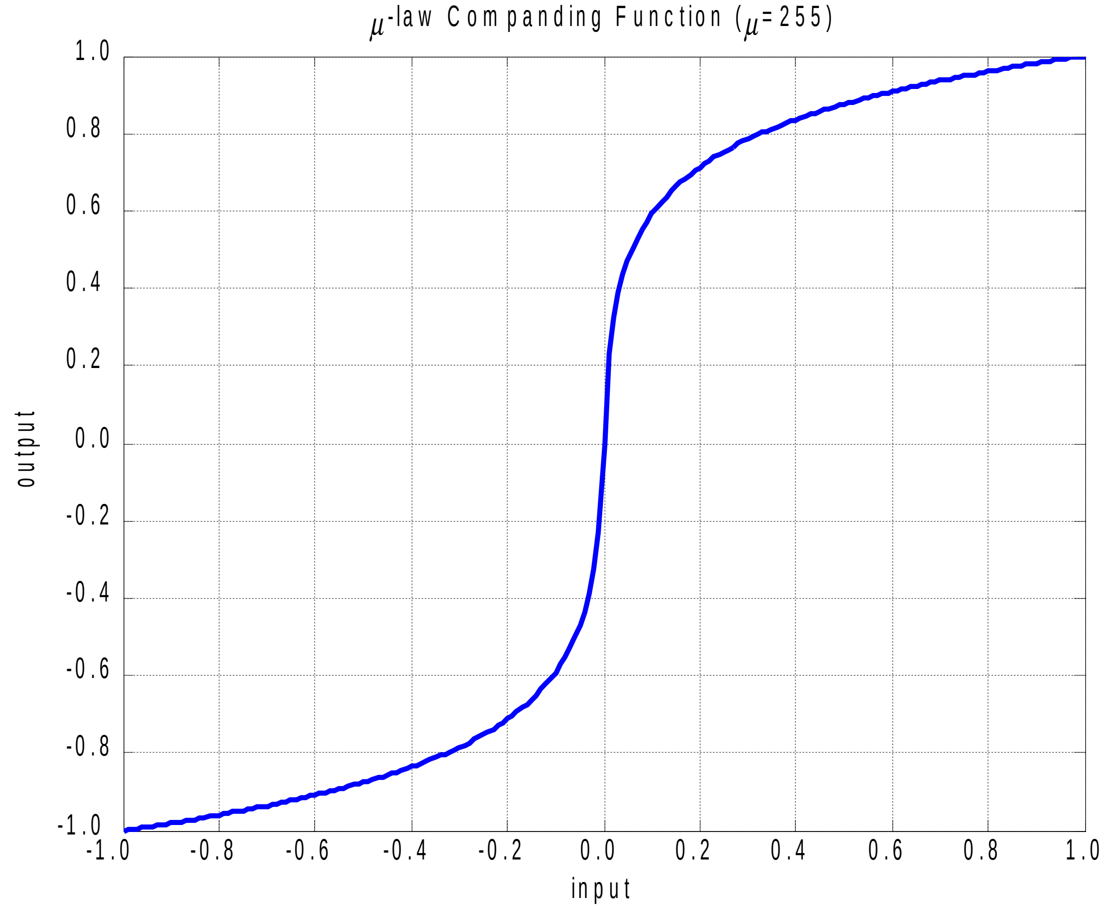
μ-law companding

- One commonly used companding function is called μ-law companding defined as

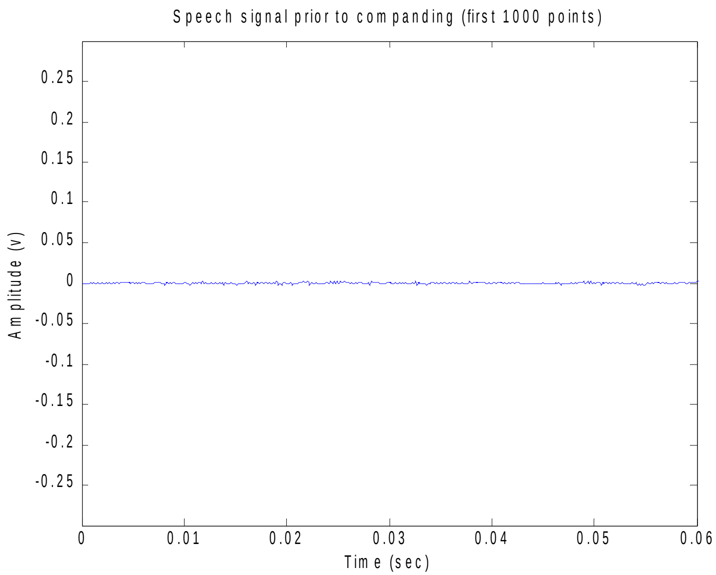
$$V_{out} = \frac{V_{\max} \times \ln(1 + \mu V_{in} / V_{\max})}{\ln(1 + \mu)}$$

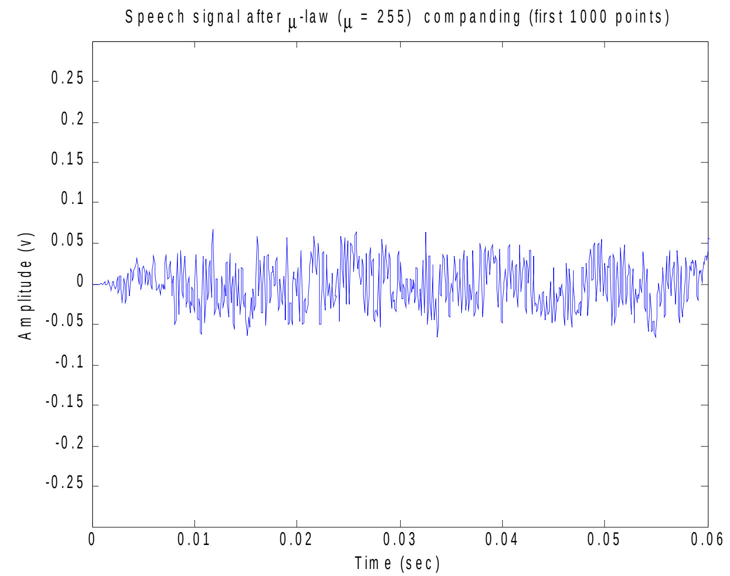
- PCM telephone systems is the U.S., Canada and Japan μ-law companding with $\mu = 255$.

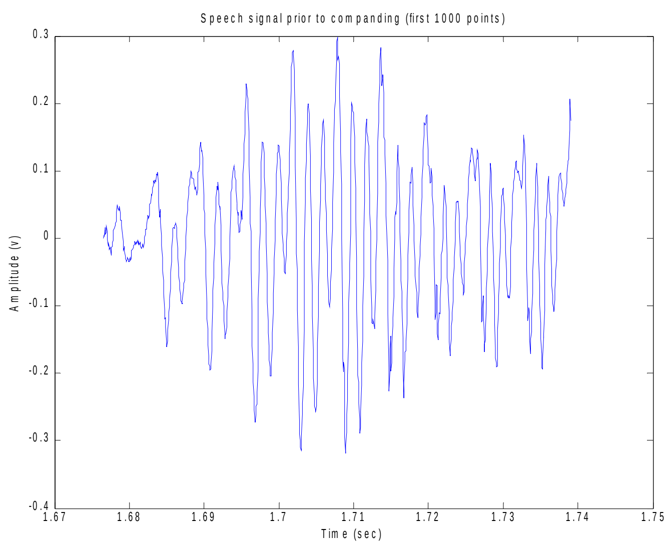
μ-law companding

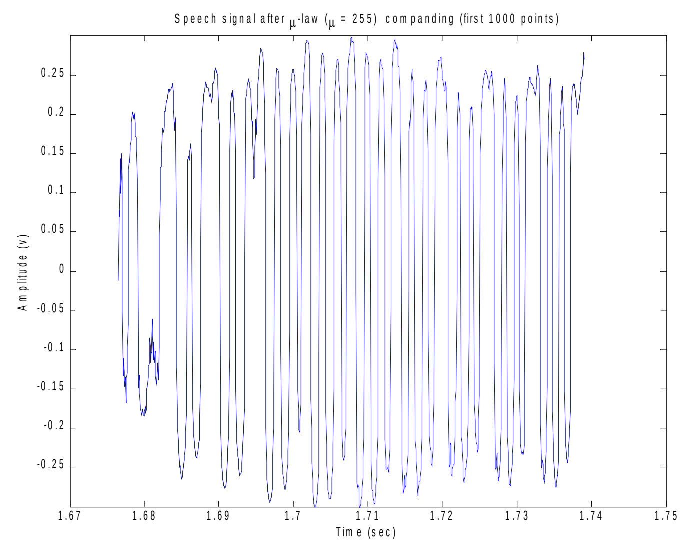


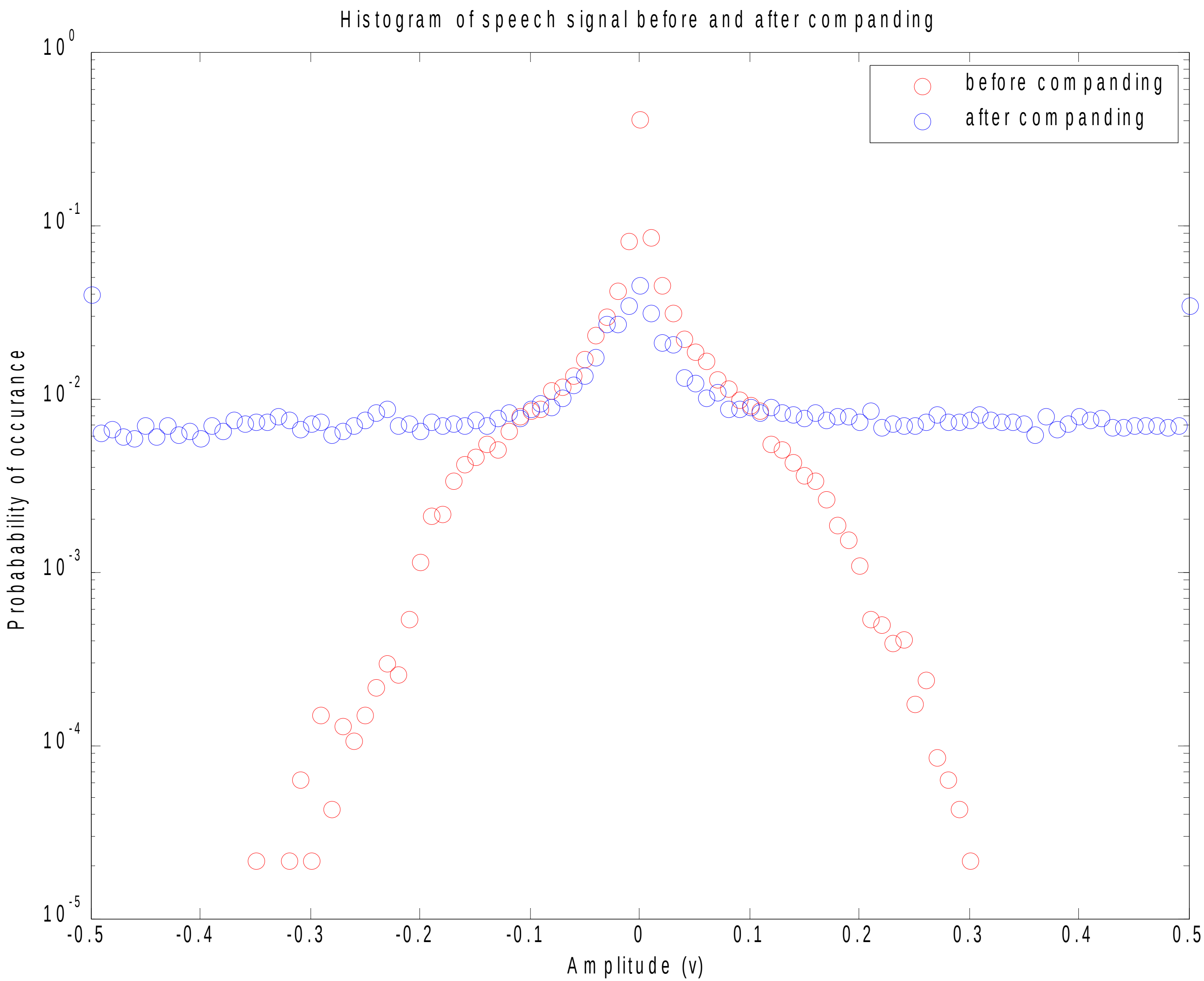
μ-law companding










Histogram of speech signal before and after companding
before companding
after companding
Probabability of occurance
Amplitude (v)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

μ-law companding

- **Bottom line**: The increase in *S/N* as a result of non-uniform quantization (through μ-law companding) allows an 8-bit non-uniform quantizer to achieve the same quality speech as a 12-bit uniform quantizer.
- For the phone company, this means a savings of 33% in required bandwidth.

Original sample file (16-bit)

Sample file after 6-bit uniform quantization

Sample file after 6-bit non-uniform quantization w/ μ=255 companding